Representational Format and Universal Quantifiers

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Formal analogs of *each*, *every*, and *all* can be defined using the tools of first-order logic as in (1-2), or in overtly second-order terms as in (3-4), which represent the quantificational content as a genuine relation that a set Y bears to a set X.

(1) $\lambda Y.\lambda X.\forall x{:}x \in Y(x \in X)$ (3) $\lambda Y.\lambda X.Y \subseteq X$

(2) $\lambda Y.\lambda X.{\sim}\exists x{:}x \in Y(x \notin X)$ (4) $\lambda Y.\lambda X.Y = Y \cap X$

Theorists often abstract away from such distinctions, focusing instead on the content shared across (1-4). Differences are treated as notational variants. But it is an empirical question whether, in speakers' minds, a word like *each* is represented as the format-neutral "shared content" of all possible specifications rather than in one particular format (Lidz et al., 2011; Pietroski et al., 2011). We offer experimental evidence that the formal distinction between first- and second-order specifications is psychologically realized and has detectable symptoms. Specifically, we argue that *each* has a first-order format whereas *every* and *all* have second-order formats, like the undeniably second-order *most* (Rescher, 1969).

In our experiments, participants were presented with quantificational statements (e.g., "each big dot is blue") and instructed to judge their truth with respect to pictures of dots of various sizes and colors. Then they were asked to guess the cardinality of a subset (e.g., "how many big dots were there?"). Our logic is that if a quantifier *Q* has a second-order meaning, the phrase *Q big dot(s)* should prompt speakers to represent the big dots taken together, in a way that promotes representations of group properties like cardinality. Therefore, quantifiers with second-order specifications should yield better performance on the relevant "how many" questions. To judge this, participants were fit with a standard psychophysical model of number estimation, which allows for comparisons of accuracy and precision (i.e., amount of variability in responses) (Odic et al., 2016).

As a proof-of-concept, experiment 1 compared *most*-statements like "most of the big dots are blue" to existential-statements like "there is a big dot that's blue". Following *most*-statements participants were more accurate ($F_{1,11}$=4.57 p<0.056; t_{11}=2.94, p<0.05) and more precise ($F_{1,11}$=8.91 p<0.05; t_{11}=3.86, p<0.01) at estimating the cardinality of the set denoted by the restrictor (e.g., big dots) than they were following existential-statements. For unmentioned (distractor) sets' cardinalities (e.g., small dots), participants showed poor performance across the board.

Experiment 2 pitted *each* and *all* against each other with statements like "each/all of the big dots are blue". Participants were more accurate ($F_{1,12}$=9.96, p<0.01; t_{12}=3.4, p<0.01) and more precise ($F_{1,12}$=3.35, p=0.092; t_{12}=3.8, p<0.01) at guessing the cardinality of the restrictor set following *all*-statements compared to *each*-statements. Experiment 3 finds the same result when comparing *every*- to *each*-statements: *every* patterns like *all* and *most* in accuracy ($F_{1,10}$=7.04, p<0.05; t_{10}=2.6, p<0.05) and precision ($F_{1,10}$=11.45, p<0.01; t_{10}=4.34, p<0.01). Experiment 4 (not pictured) directly compared *every*-statements with *all*-statements. Here we find no significant effects, as performance was nearly identical across blocks. We take the results of experiments 1-4 as evidence that *each* has a first-order format whereas *every* and *all* have second-order formats. More broadly, our results support the idea that meanings are not format-neutral, but are mentally represented in particular and detectable ways.