The psychological treatment of (universal) quantification

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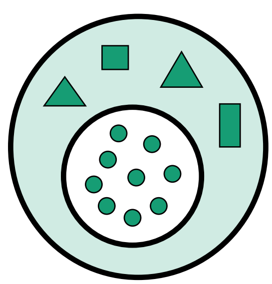
UMD Linguistics
11.22.19 Penn ILST

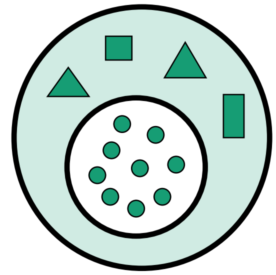
1

⟦**Each / Every circle is green**⟧ =



(e.g, Barwise & Cooper 1981)

2



⟦**Each / Every circle is green**⟧ = TRUE *iff*

$\forall x : \text{circle}(x)[\text{green}(x)]$

≈ for each thing that's a circle, it's green

$\neg\exists x : \text{circle}(x)[\neg\ \text{green}(x)]$

≈ there's no thing that's a circle but not green

$\text{CIRCLES} \subseteq \text{GREEN-THINGS}$

≈ the set of circles is a subset of the set of green things

$\text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS}$

≈ the set of circles is identical to the set of green circles

$\exists\text{CIRCLES} \ \&\ \forall x{:}x \in \text{CIRCLES}[\text{green}(x)]$

≈ the members of the set of circles are each green

3

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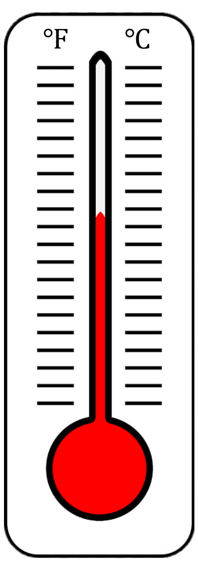

"…I believe that the model-theoretic intension of a word has in principle nothing whatsoever to do with what goes on in a person's head when he uses that word"

-Dowty (1979)

4

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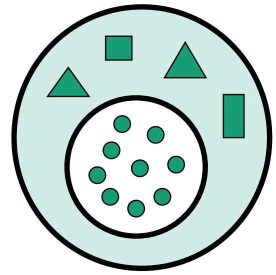

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CIRCLES ⊆ GREEN-THINGS

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CIRCLES = CIRCLES ∩ GREEN-THINGS

≈ the set of circles is identical to the set of green circles

∃CIRCLES & $\forall x : x \in$ CIRCLES[green(x)]

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5

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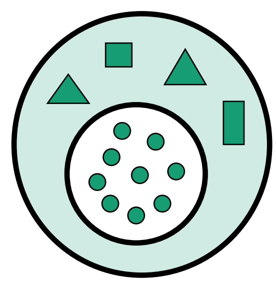

First-order (categorizing individuals)

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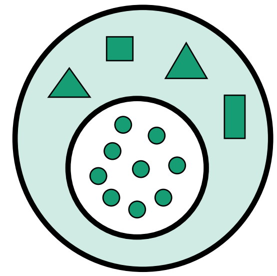

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Second-order (implicating groups)

$\text{CIRCLES} \subseteq \text{GREEN-THINGS}$
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7

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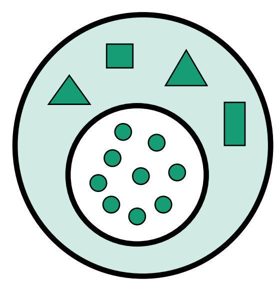

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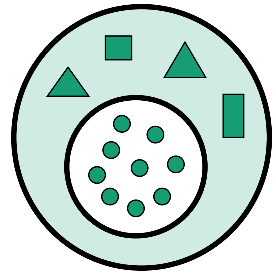

First-order
(categorizing individuals)

$\forall x : circle(x)[green(x)]$
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Second-order
(implicating groups)

Each is mandatorily distributive (Vendler, 1962; Dowty, 1987)

*Each (of the) student(s) ?Every student All of the students	met at the bar / gathered

9

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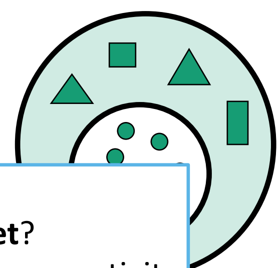

First-order
(categorizing individuals)

Does *every*'s meaning relate **2 sets** or
call for first-order quantification relativized to **1 set**?
→ The latter suggests a semantic account of conservativity

Second-order
(implicating groups)

$\boxed{2}$
CIRCLES ⊆ GREEN-THINGS
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CIRCLES = CIRCLES ∩ GREEN-THINGS
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$\boxed{1}$
∃CIRCLES & ∀x:x ∈ CIRCLES[green(x)]
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10

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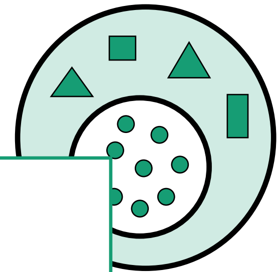

First-order (categorizing individuals)

Second-order (implicating groups)

Bigger picture questions:

→ Are lexical meanings invariant across people?

→ Are they structured or atomic?

→ If invariant & structured, how are they acquired?

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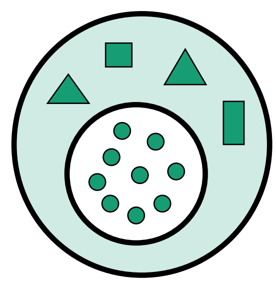

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Second-order (implicating groups)

- CIRCLES $\subseteq$ GREEN-THINGS
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- $\exists$CIRCLES & $\forall x{:}x \in$ CIRCLES[green(x)]
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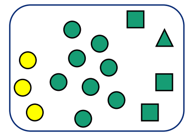

"Each/Every circle is green"

Categorizing individuals

meaning → $\forall x : \text{circle}(x)[\text{green}(x)]$

Relating groups

CIRCLES ⊆ GREEN-THINGS

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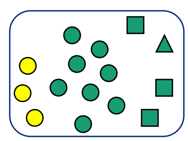
Interface transparency
(Pietroski et al. 2009; Lidz et al. 2011)

Bias to use verification strategy that directly implements meaning



"Each/Every circle is green"

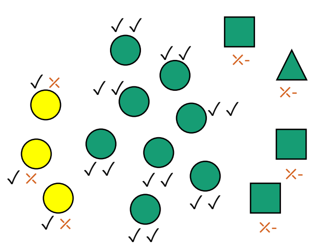
Categorizing individuals

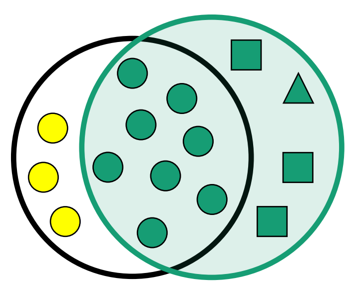
meaning → $\forall x : \text{circle}(x)[\text{green}(x)]$

Relating groups

CIRCLES ⊆ GREEN-THINGS

strategy →





14

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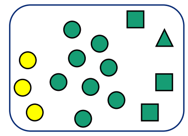


"Each/Every circle is green"

meaning →

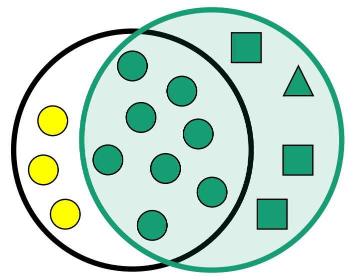
Categorizing individuals
∀x : circle(x)[green(x)]

Relating groups
CIRCLES ⊆ GREEN-THINGS

strategy →

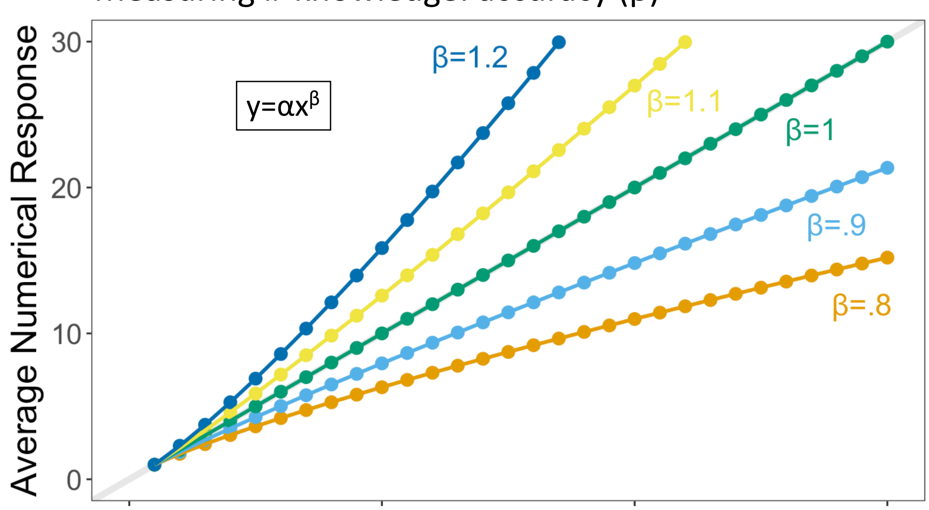
Representing groups:
good estimate of summary statistics
(**number**, average size, center of mass, …)

Ariely 2001; Cohng & Treisman 2003; Feigenson et al. 2004; Burr & Ross 2008; Alvarez 2011; ao



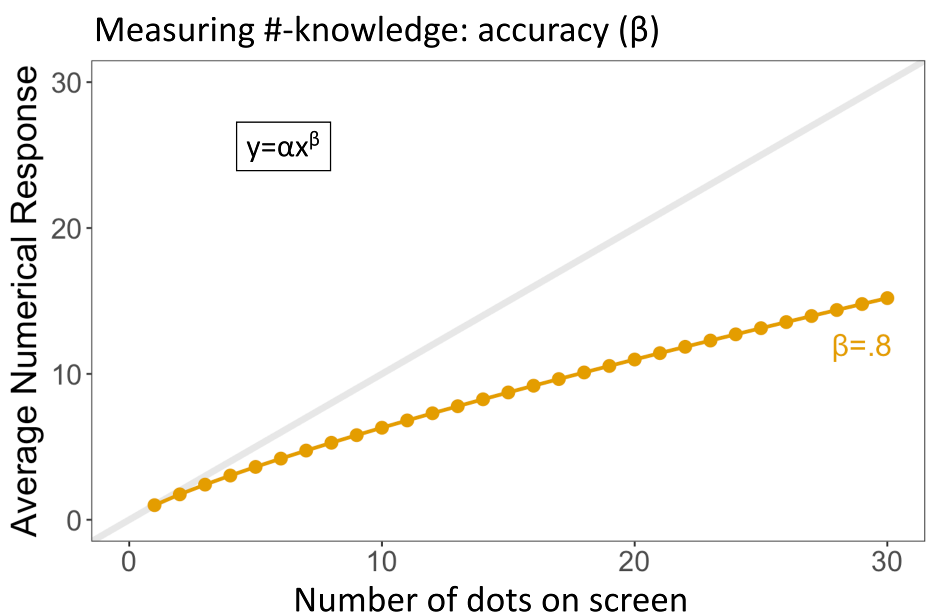
15

Measuring #-knowledge: accuracy (β)



(Stevens 1964; Krueger 1984; Odic et al. 2016; etc.)

16



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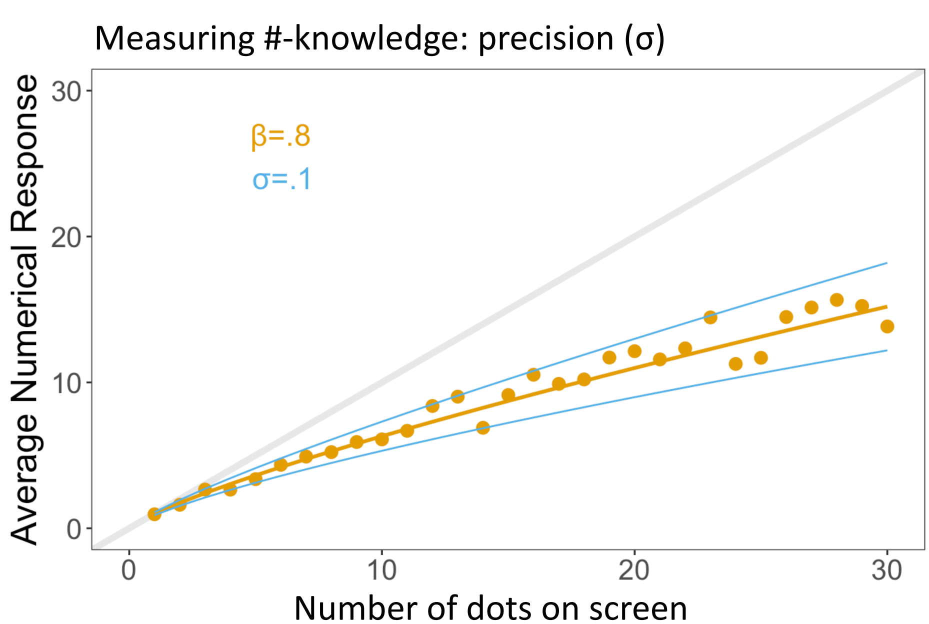
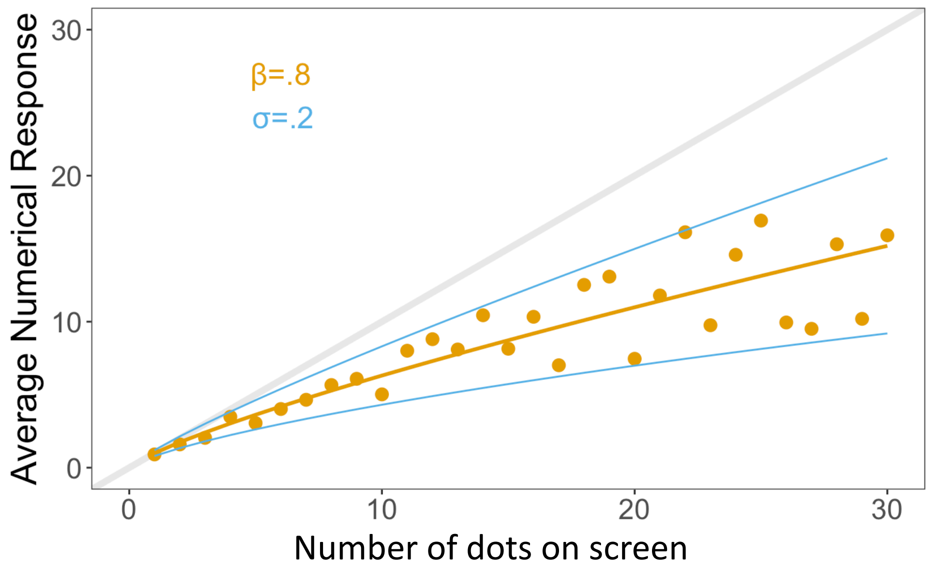
17



(Laming 1997; Odic et al. 2016; etc.)

18

Measuring #-knowledge: precision (σ)



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19

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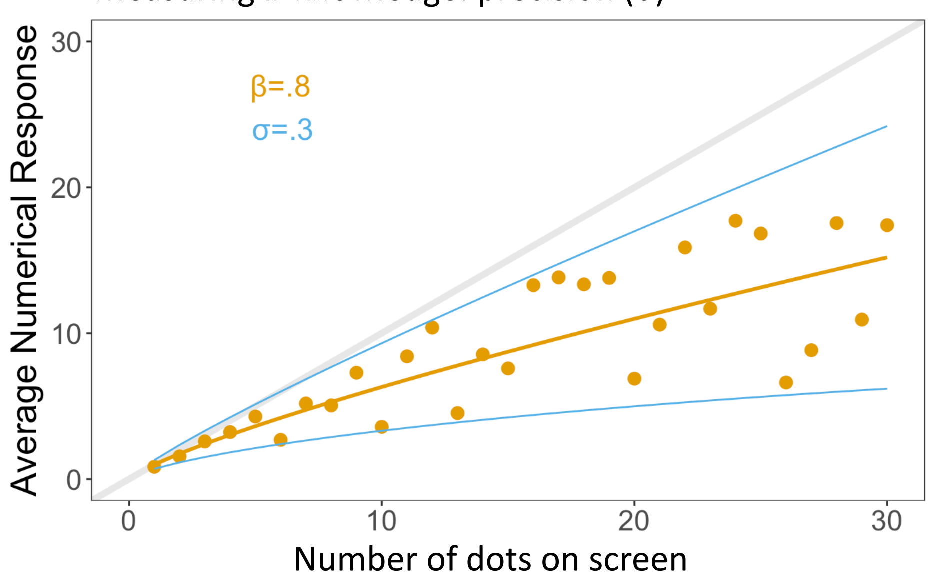

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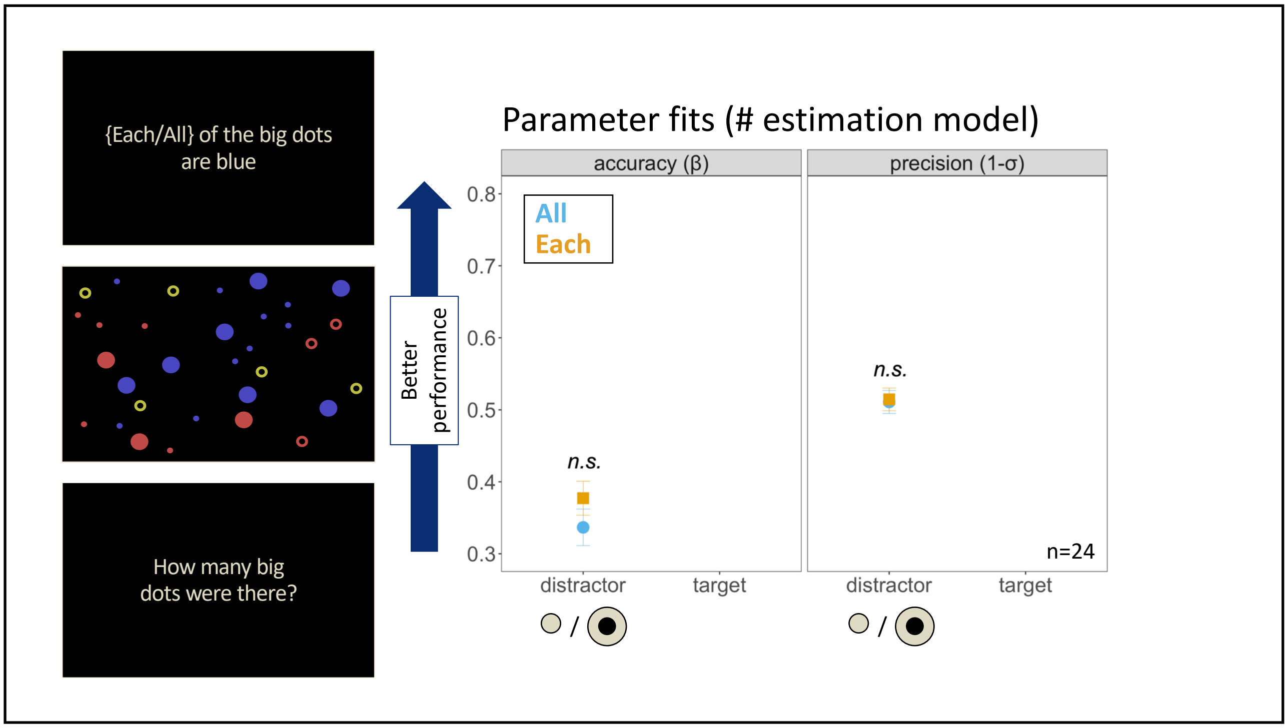
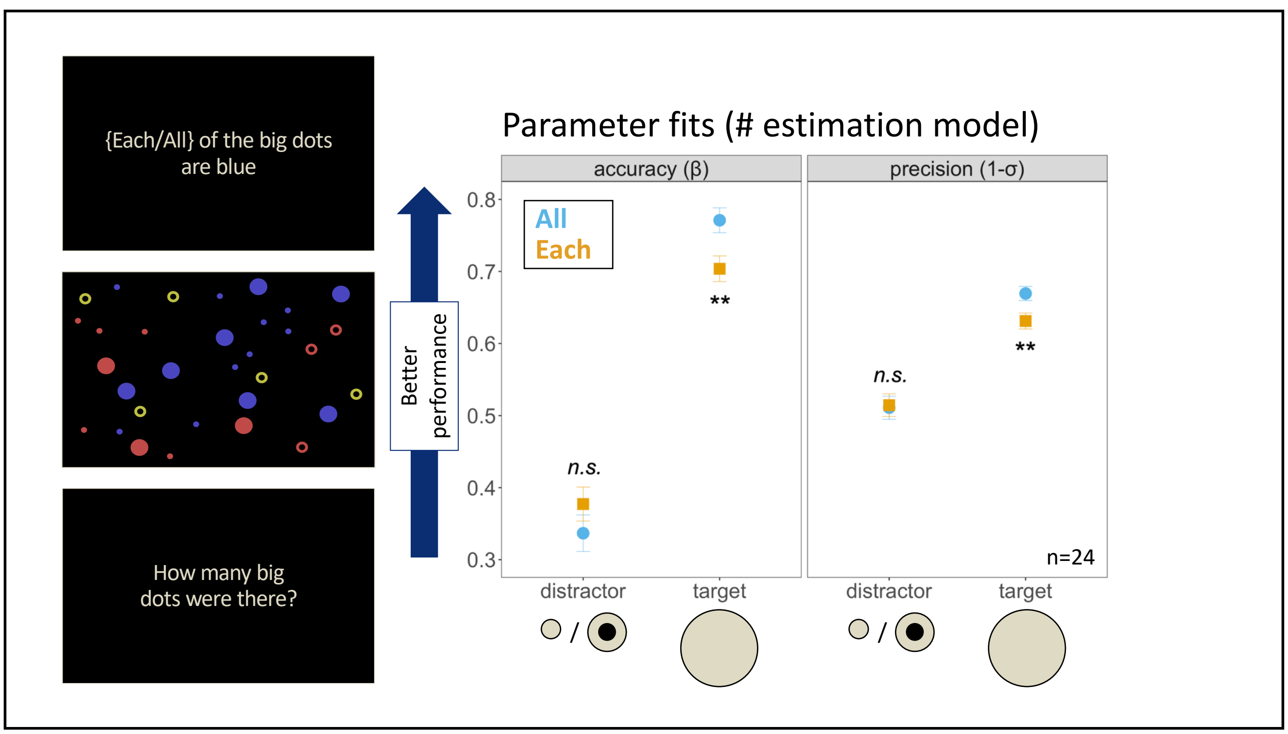
20


{Each/All} of the big dots are blue
How many big dots were there?
Better performance
Parameter fits (# estimation model)
accuracy (β)
precision (1-σ)
All
Each
0.8
0.7
0.6
0.5
0.4
0.3
n.s.
n.s.
n=24
distractor
target
distractor
target
/
/

21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n.s.
**
n.s.
**
n=24
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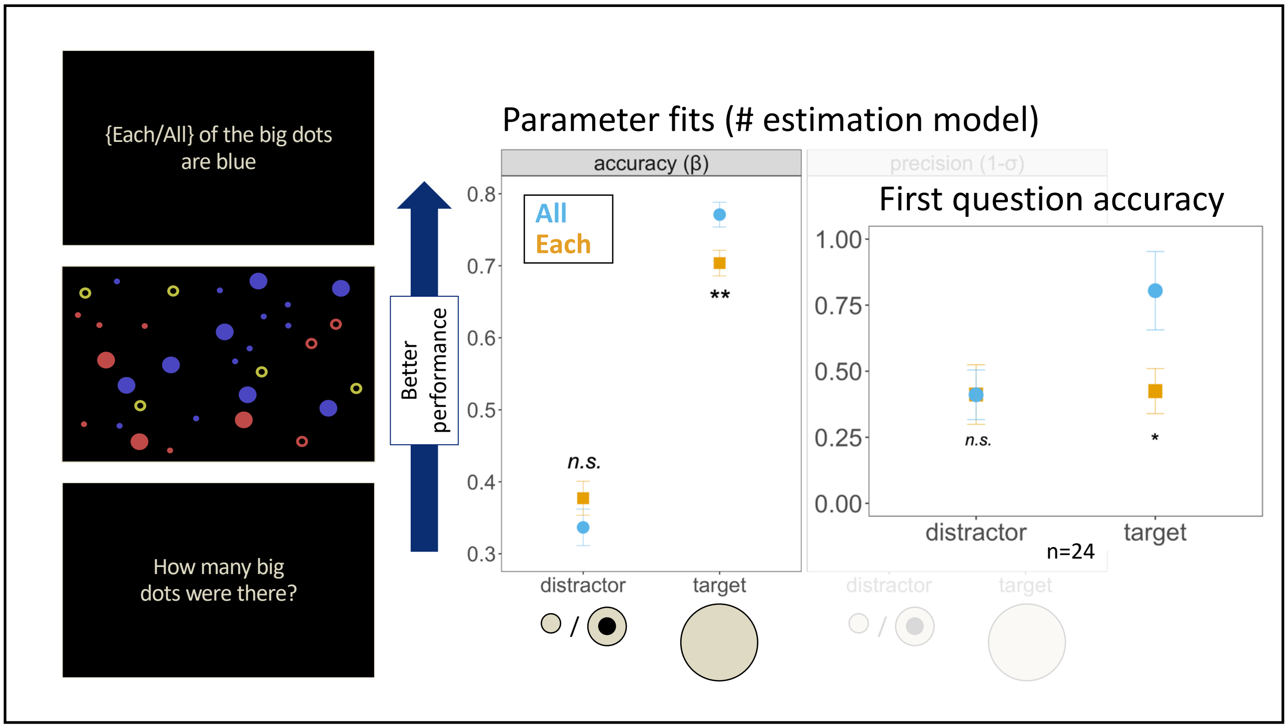
/
/

22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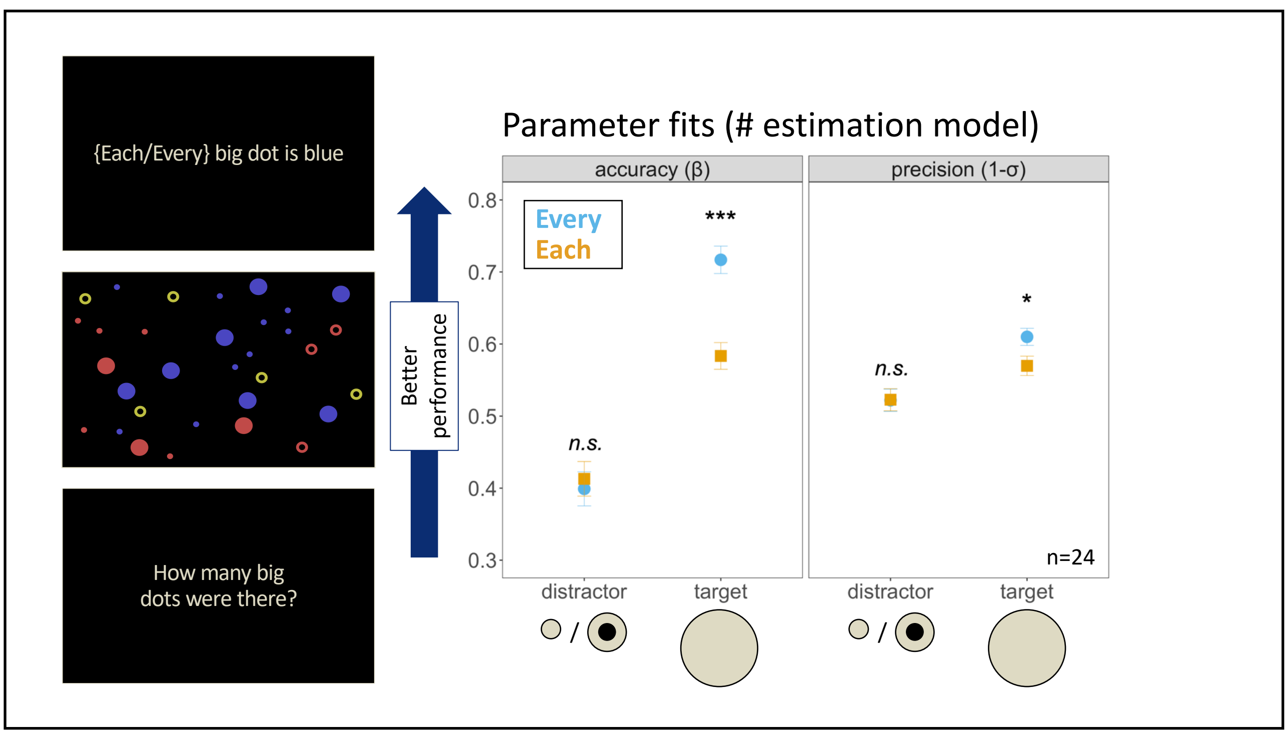
0.7
0.6
0.5
0.4
0.3
All
Each
**
n.s.
distractor
target
First question accuracy
1.00
0.75
0.50
0.25
0.00
n.s.
*
distractor
target
n=24

23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Every
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n.s.
*
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n=24
distractor
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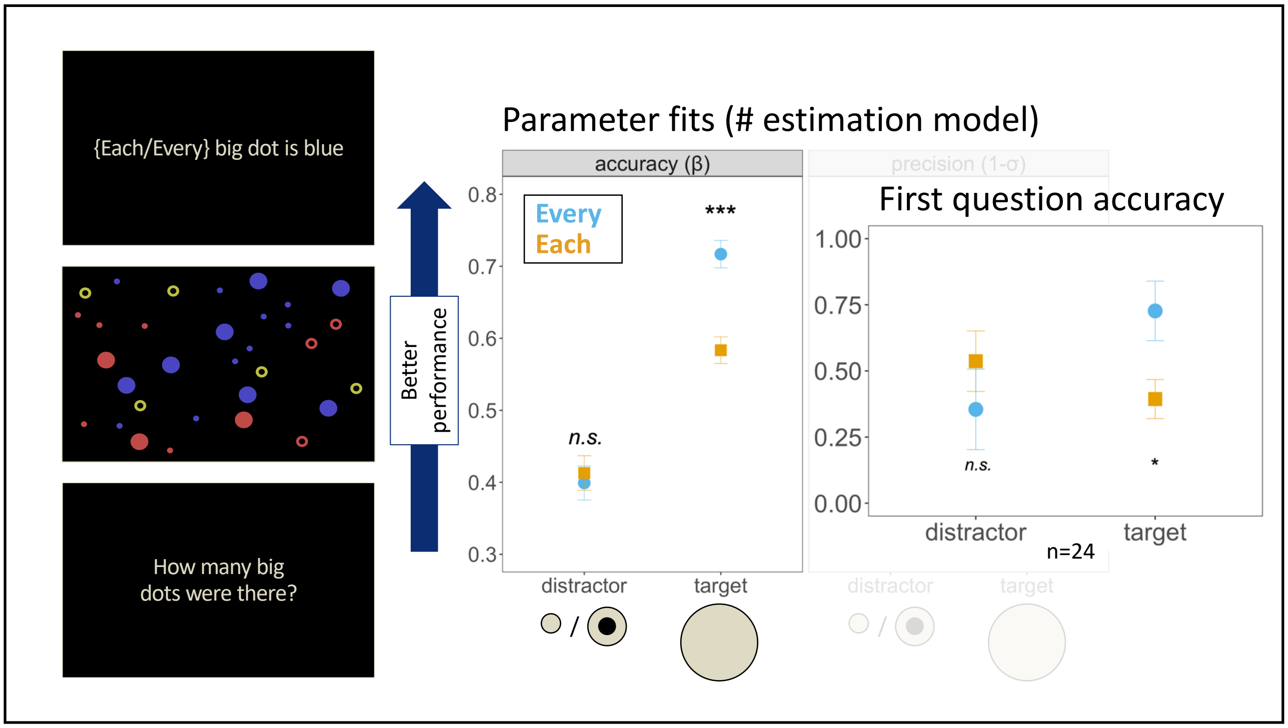
distractor
target

24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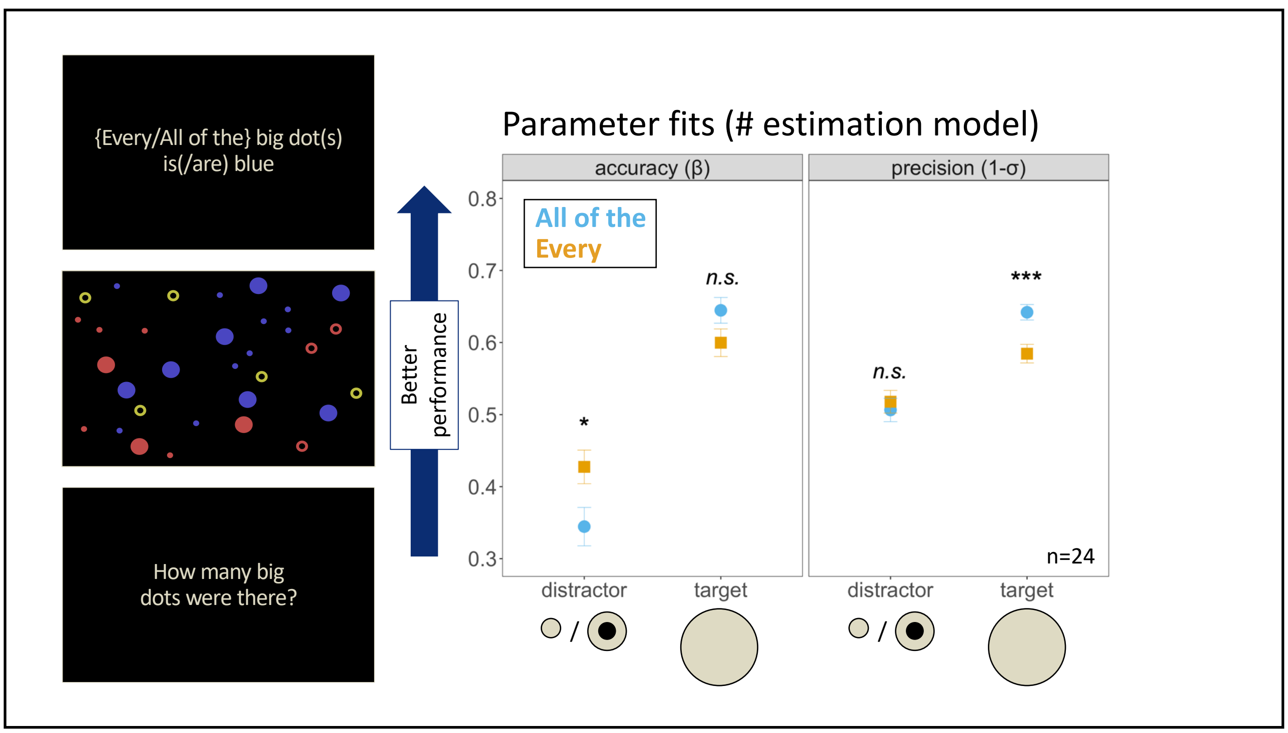
/

25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All of the
Every
n.s.
*
***
0.8
0.7
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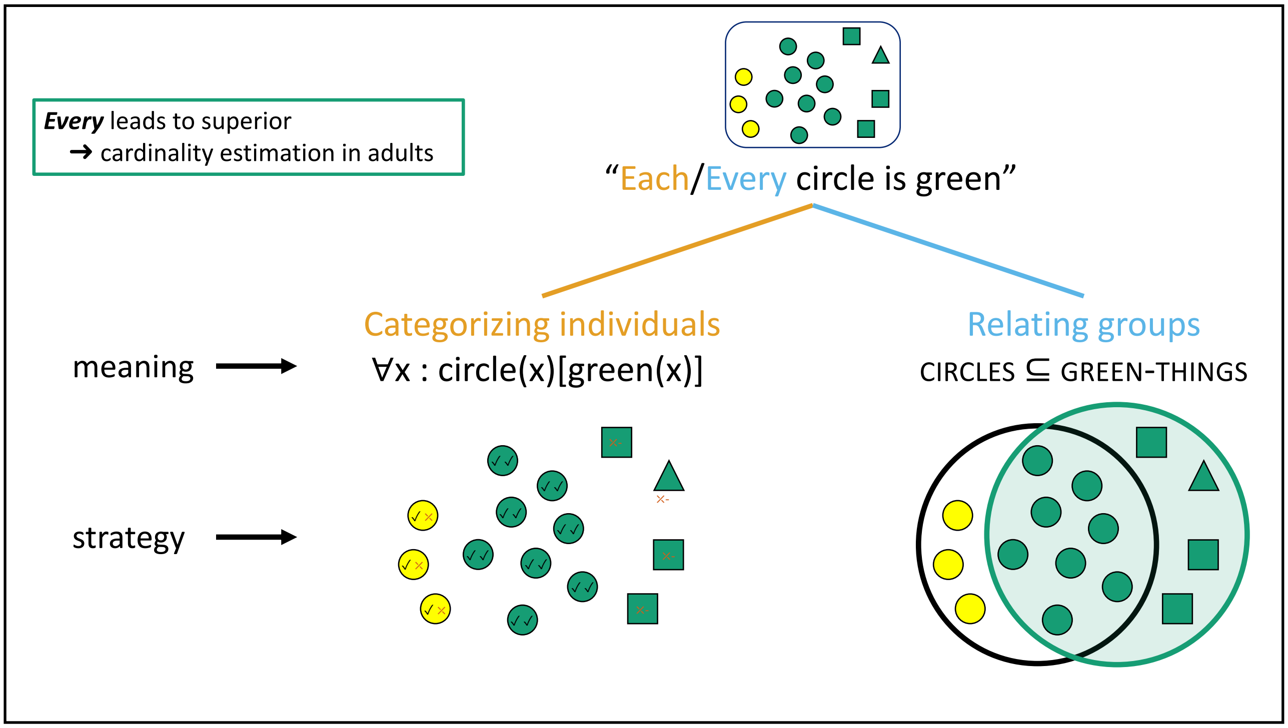
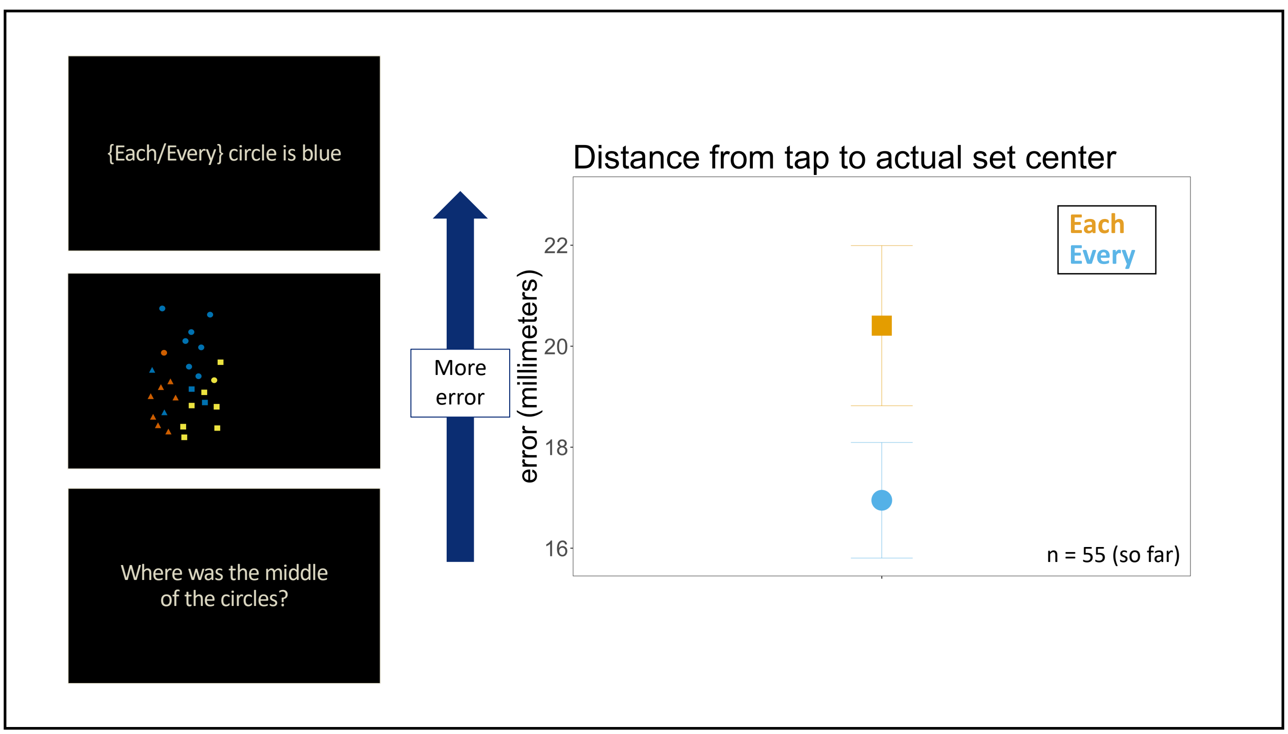
/

26



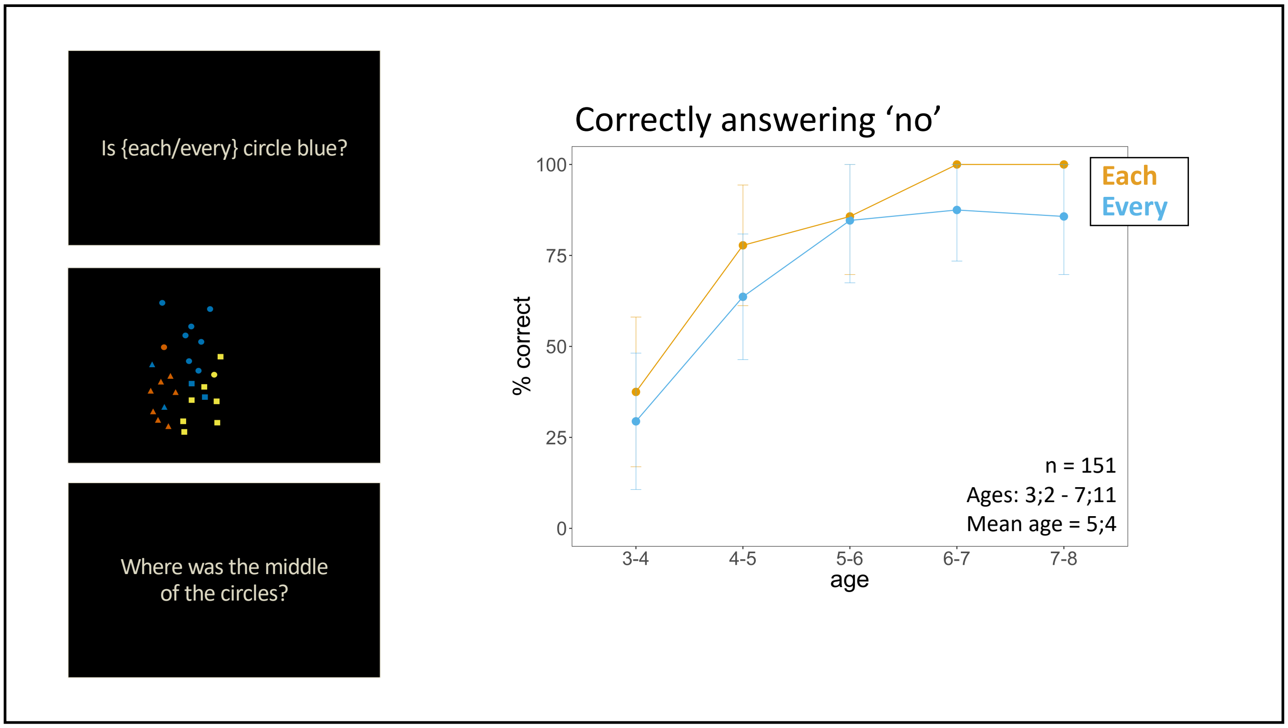
Every leads to superior
→ cardinality estimation in adults
"Each/Every circle is green"
Categorizing individuals
∀x : circle(x)[green(x)]
Relating groups
CIRCLES ⊆ GREEN-THINGS
meaning
strategy

27

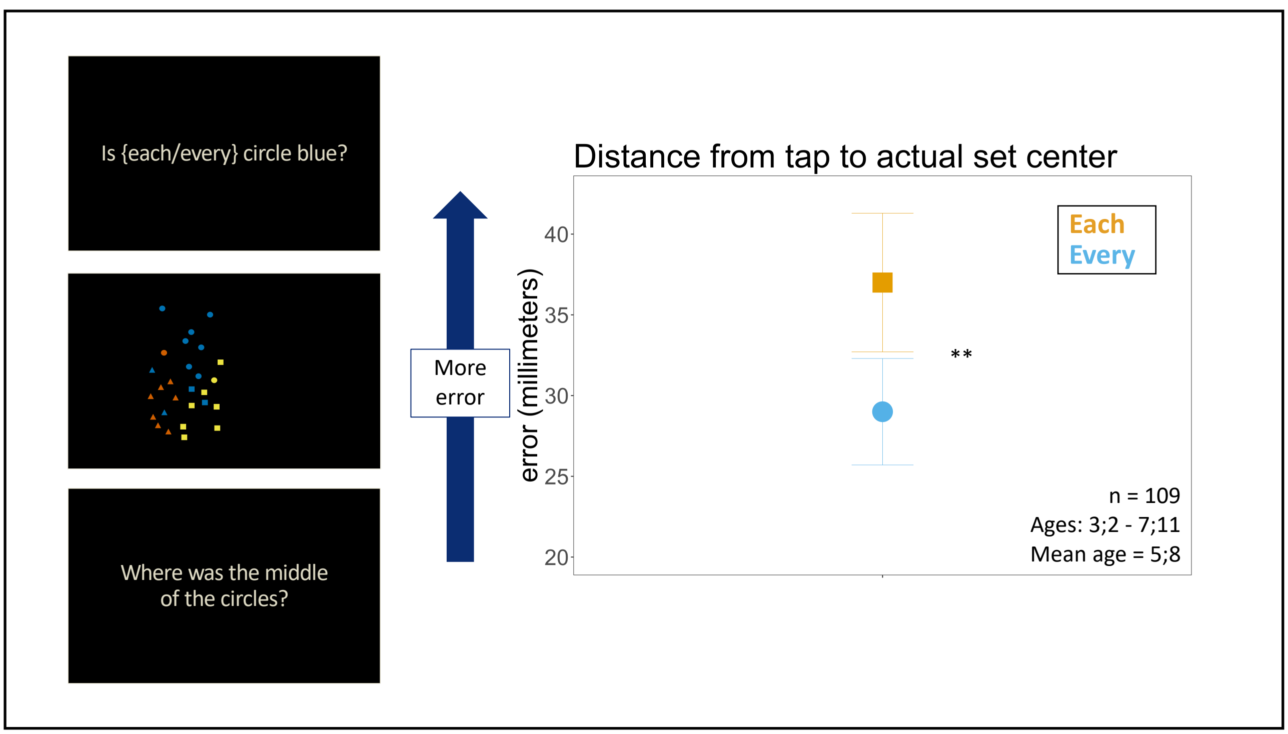


{Each/Every} circle is blue
Where was the middle of the circles?
More error
Distance from tap to actual set center
error (millimeters)
22
20
18
16
Each
Every
n = 55 (so far)

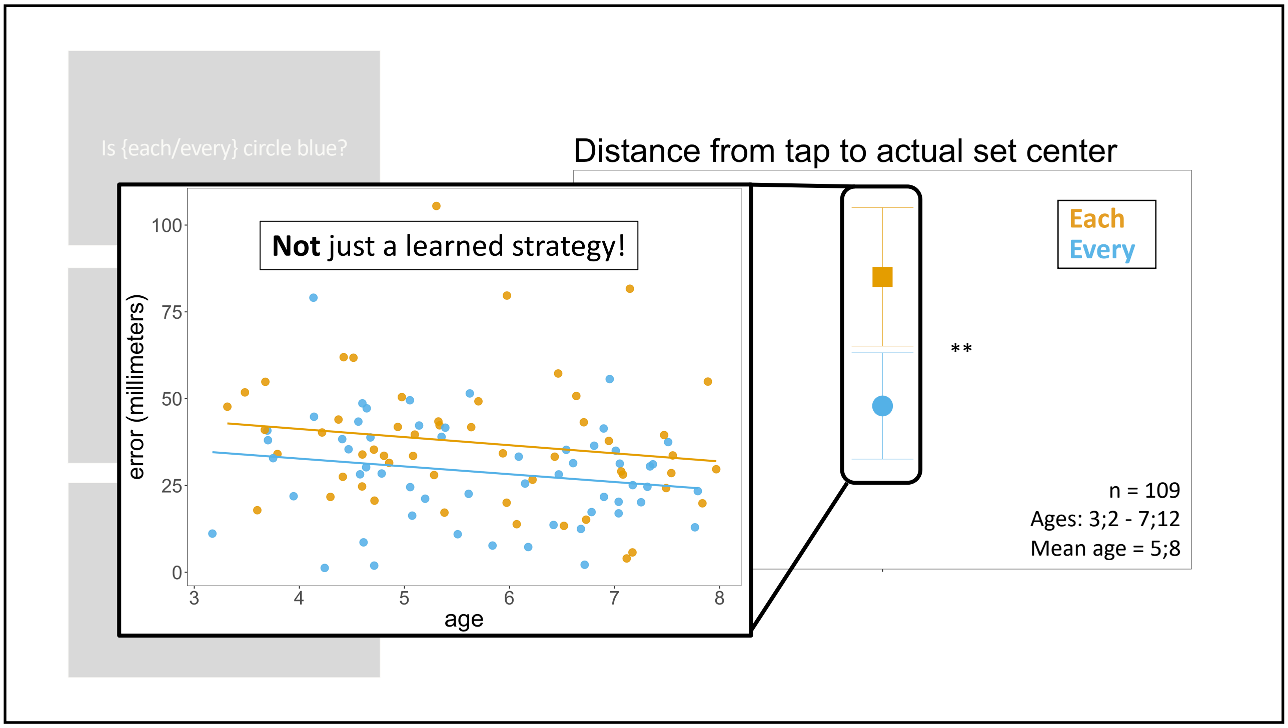
28


Is {each/every} circle blue?
Where was the middle of the circles?
Correctly answering 'no'
Each
Every
% correct
100
75
50
25
0
3-4
4-5
5-6
6-7
7-8
age
n = 151
Ages: 3;2 - 7;11
Mean age = 5;4

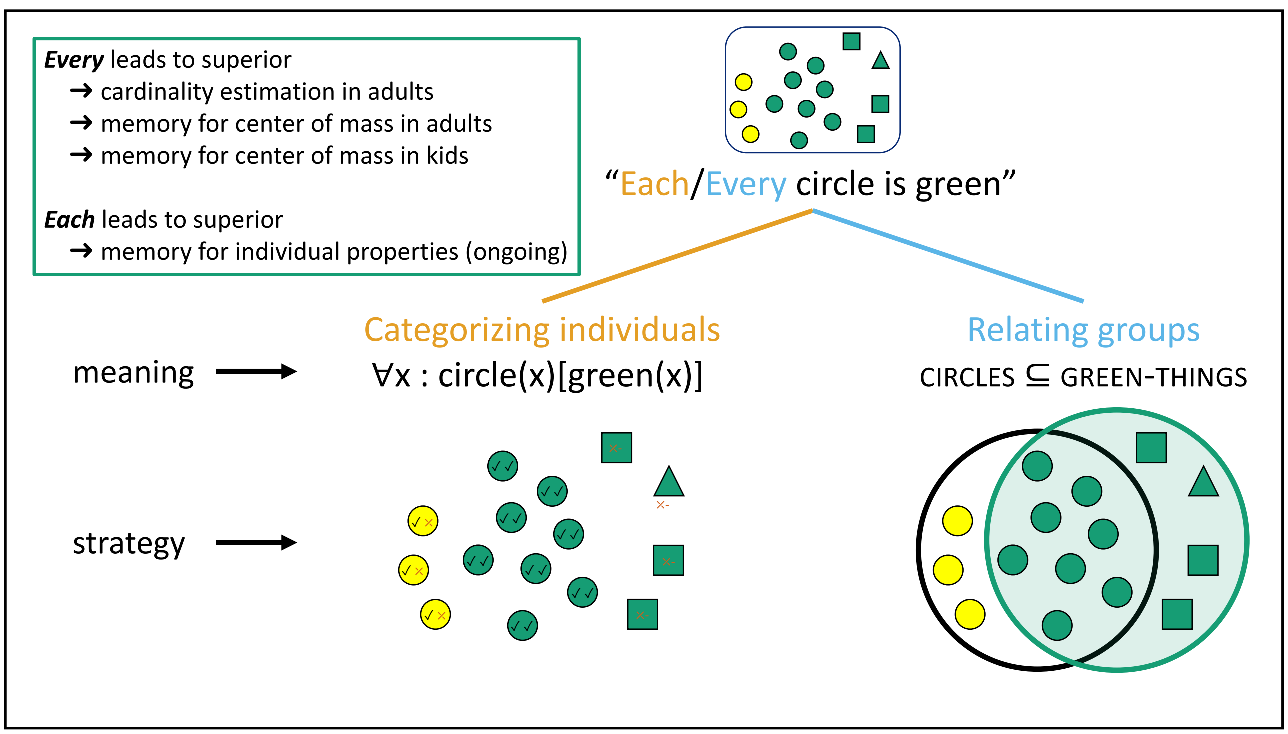
29


Is {each/every} circle blue?
Where was the middle of the circles?
More error
Distance from tap to actual set center
Each
Every
error (millimeters)
40
35
30
25
20
**
n = 109
Ages: 3;2 - 7;11
Mean age = 5;8

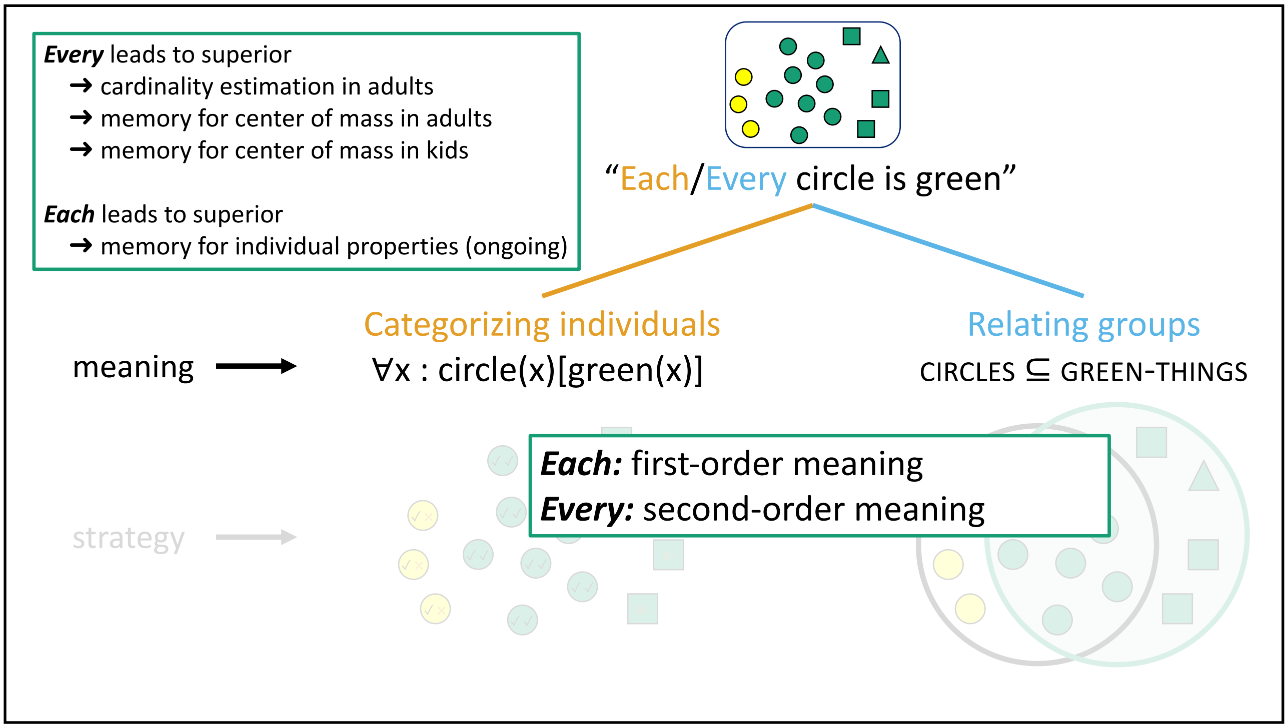
30


Is {each/every} circle blue?
Distance from tap to actual set center
Not just a learned strategy!
100
75
50
25
0
error (millimeters)
3
4
5
6
7
8
age
Each
Every
**
n = 109
Ages: 3;2 - 7;12
Mean age = 5;8

31


Every leads to superior
→ cardinality estimation in adults
→ memory for center of mass in adults
→ memory for center of mass in kids
Each leads to superior
→ memory for individual properties (ongoing)
"Each/Every circle is green"
Categorizing individuals
∀x : circle(x)[green(x)]
Relating groups
CIRCLES ⊆ GREEN-THINGS
meaning
strategy

32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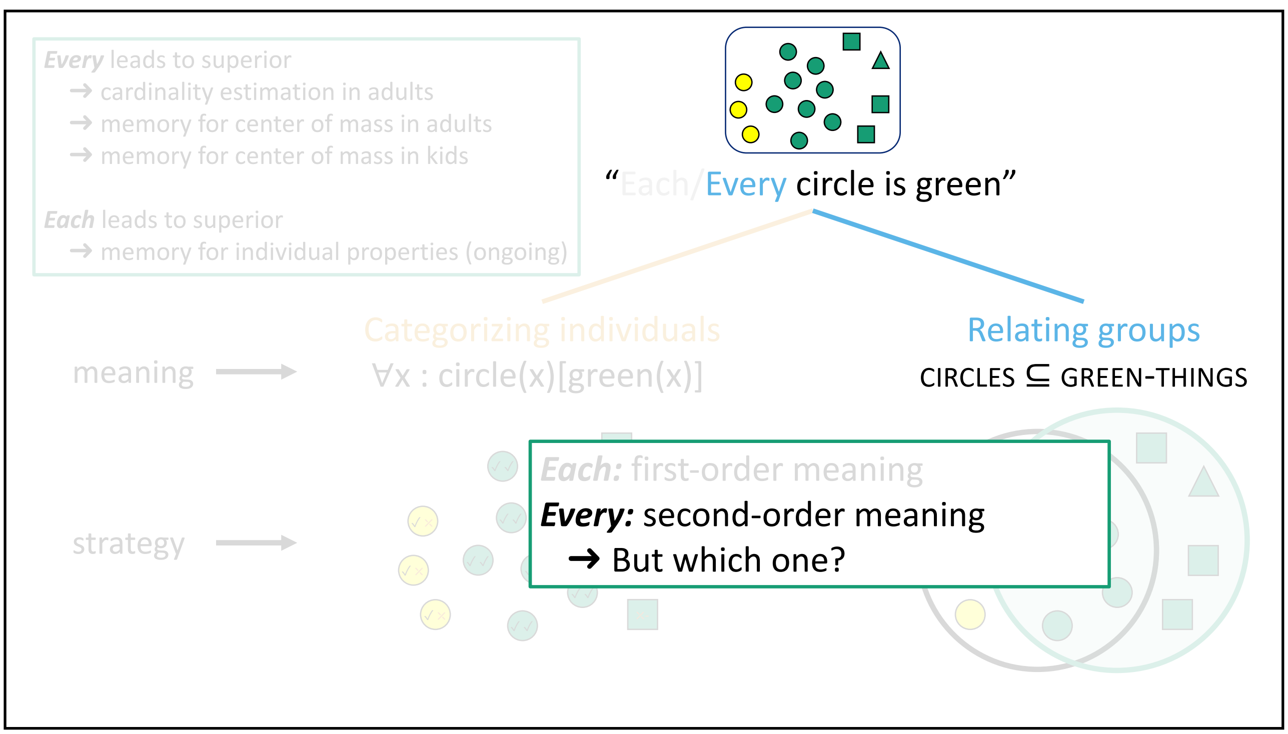
Relating groups
CIRCLES ⊆ GREEN-THINGS
meaning
strategy
Each: first-order meaning
Every: second-order meaning

33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"Each/Every circle is green"
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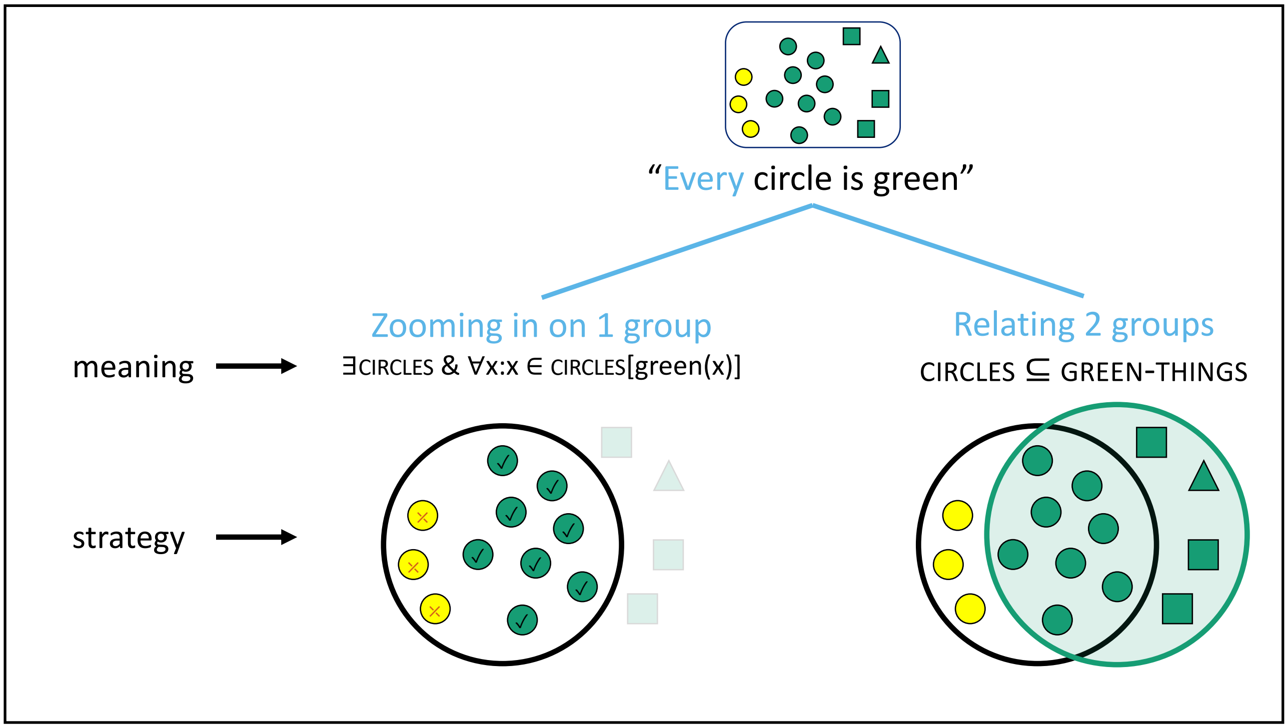
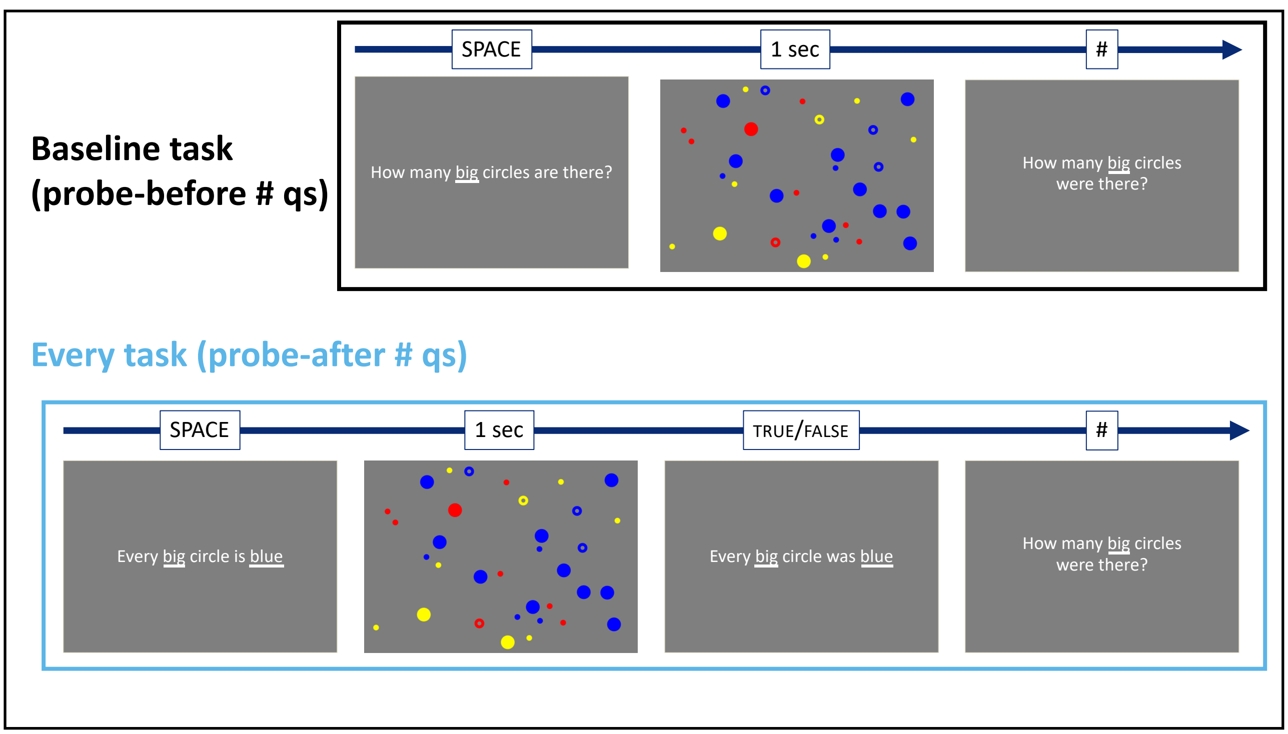
∀x : circle(x)[green(x)]
Relating groups
CIRCLES ⊆ GREEN-THINGS
meaning
strategy
Each: first-order meaning
Every: second-order meaning
→ But which one?

34



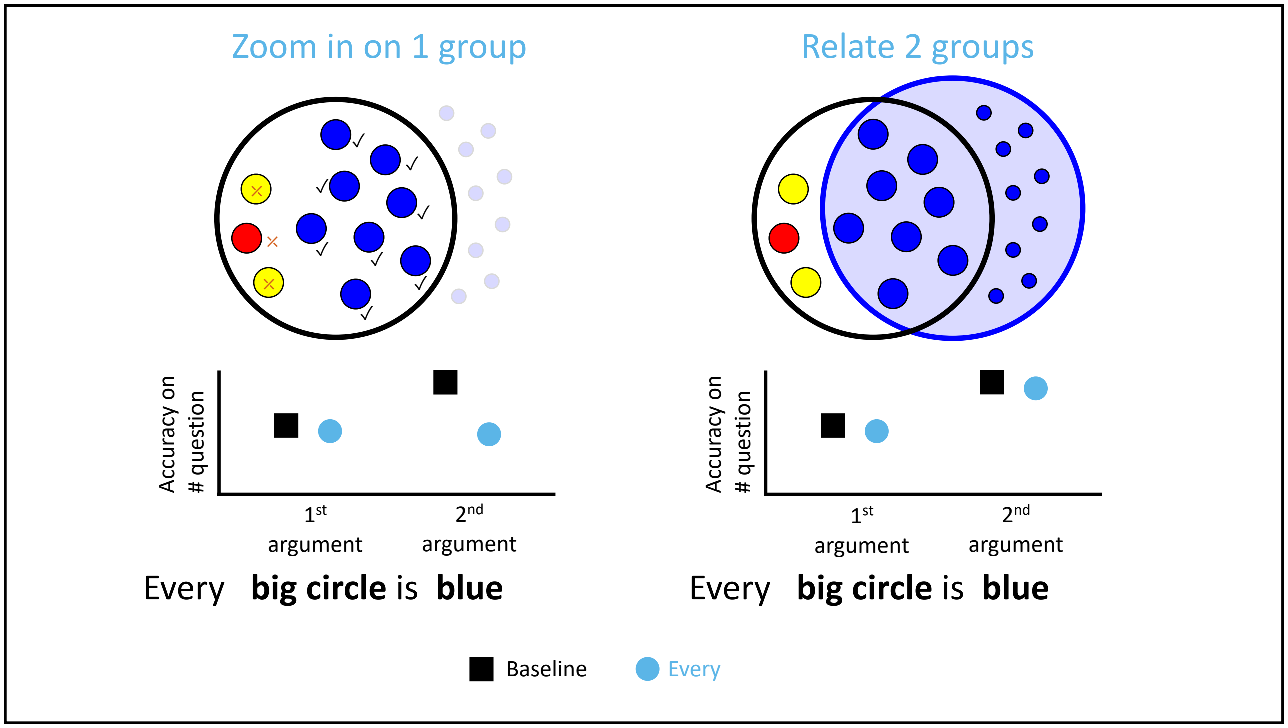
"Every circle is green"
Zooming in on 1 group
∃CIRCLES & ∀x:x ∈ CIRCLES[green(x)]
Relating 2 groups
CIRCLES ⊆ GREEN-THINGS
meaning
strategy

35



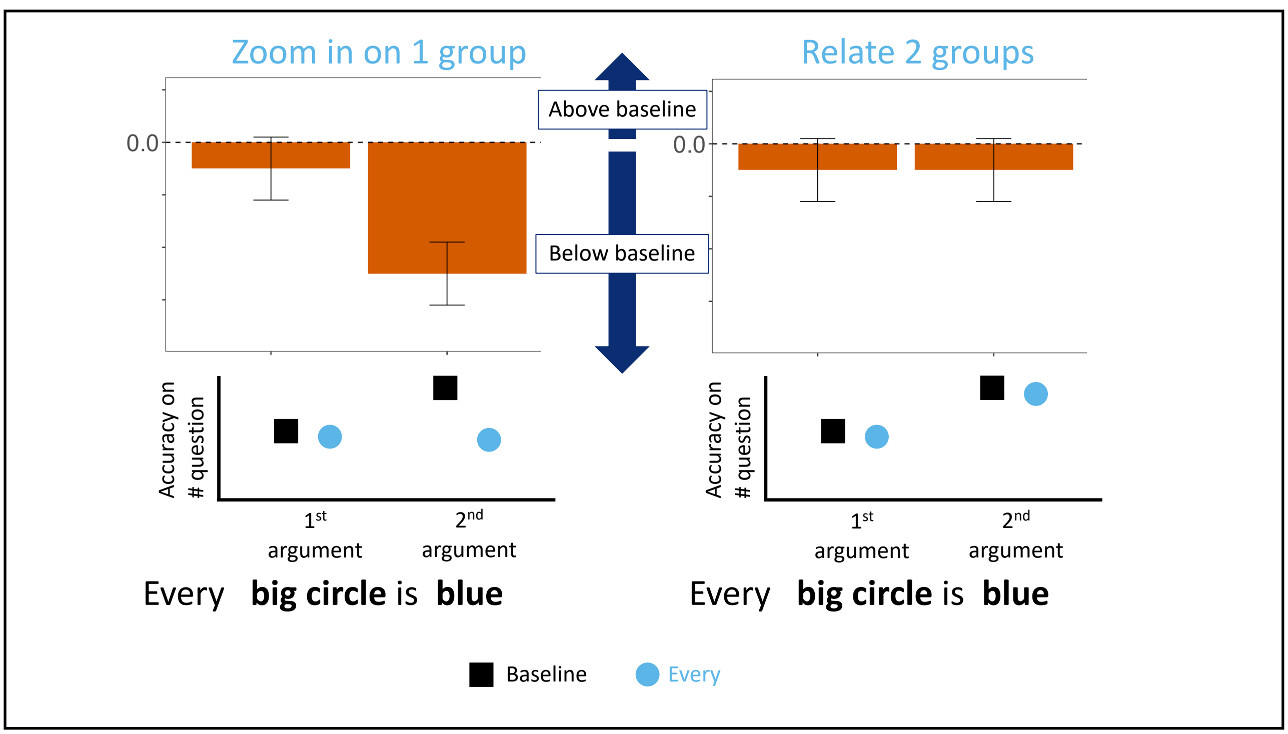
Baseline task (probe-before # qs)
SPACE
1 sec
#
How many big circles are there?
How many big circles were there?
Every task (probe-after # qs)
SPACE
1 sec
TRUE/FALSE
#
Every big circle is blue
Every big circle was blue
How many big circles were there?

36



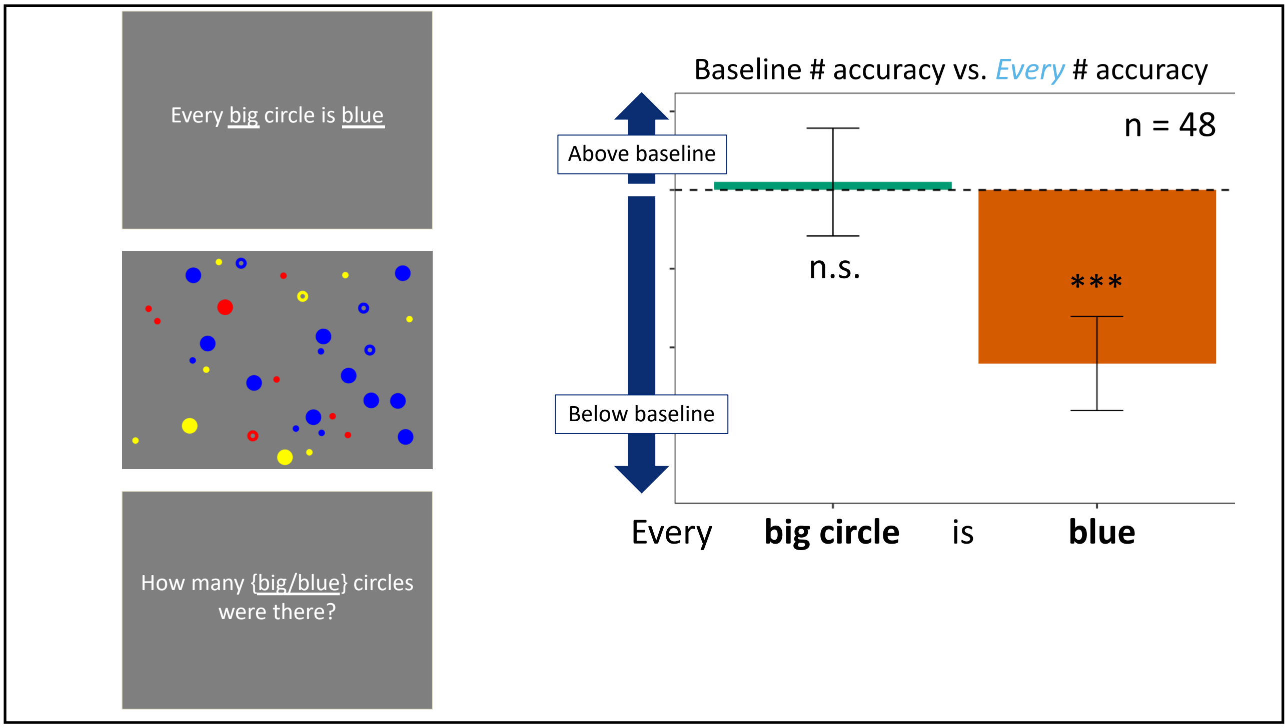
Zoom in on 1 group
Relate 2 groups
Accuracy on # question
1st argument
2nd argument
Every big circle is blue
Baseline
Every

37

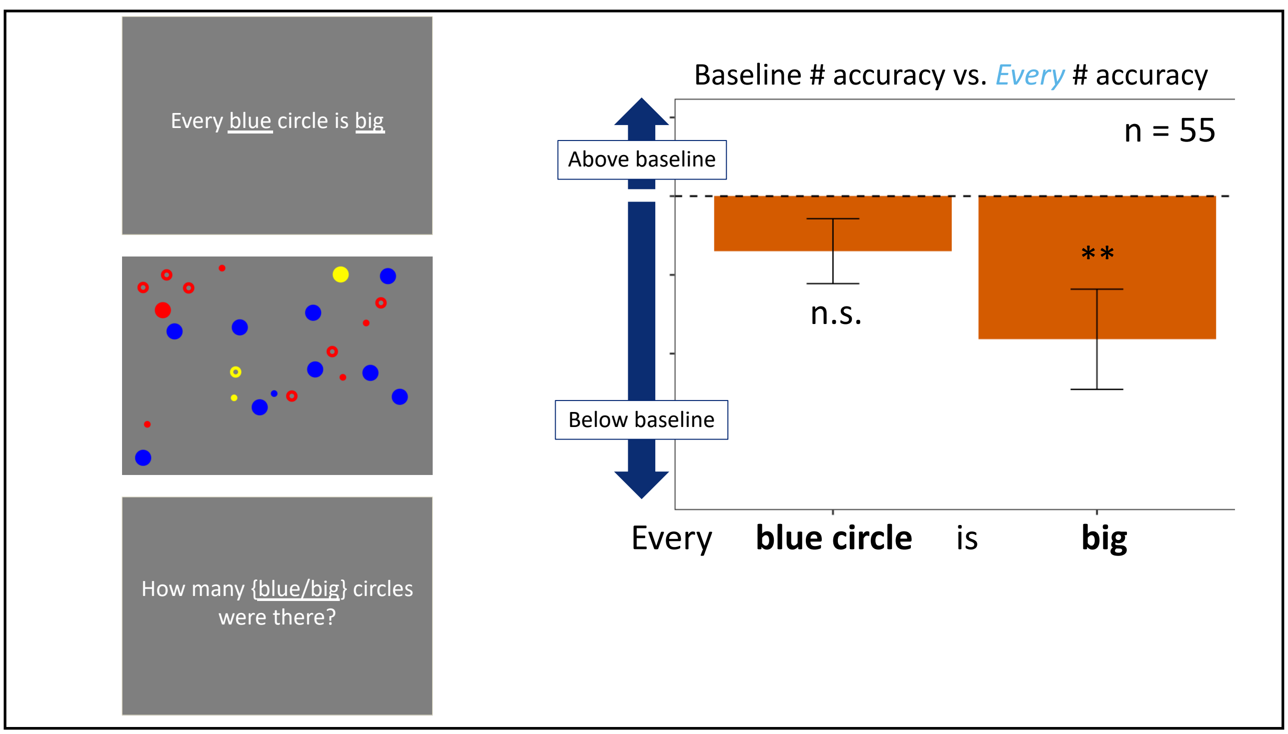


Zoom in on 1 group
Relate 2 groups
0.0
Above baseline
Below baseline
Accuracy on # question
1st argument
2nd argument
Every big circle is blue
Baseline
Every

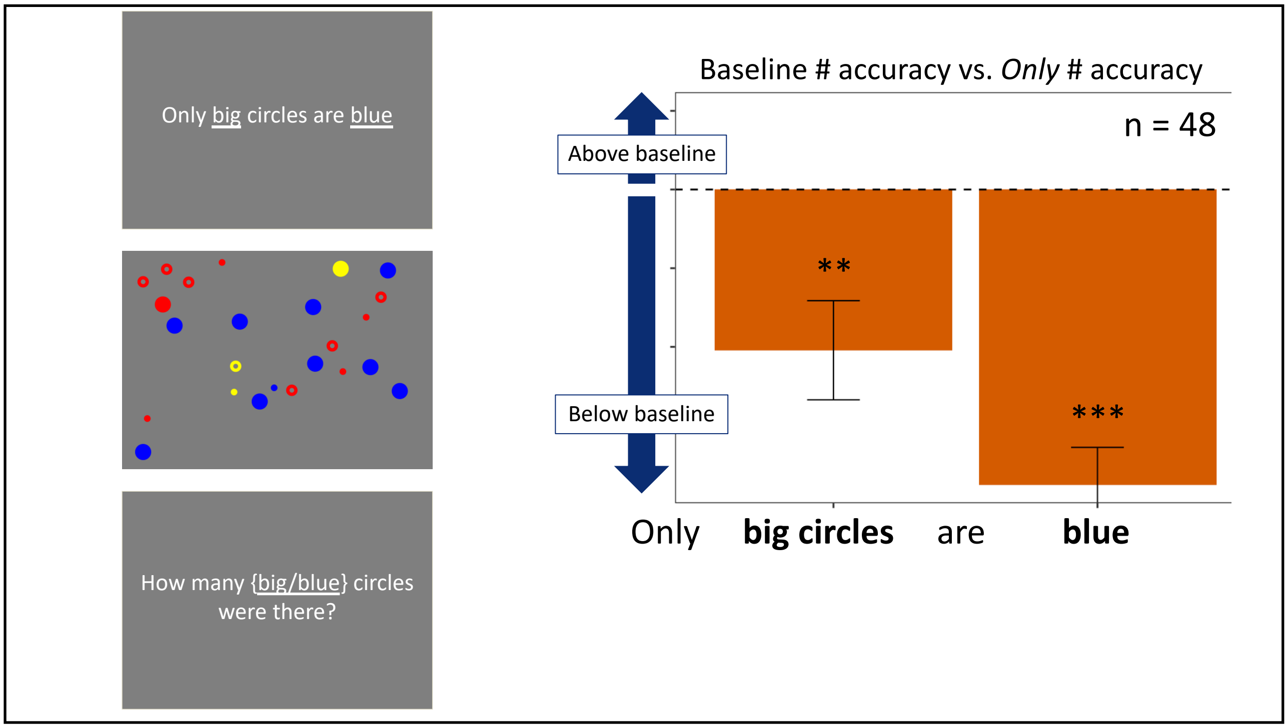
38


Every big circle is blue
How many {big/blue} circles were there?
Baseline # accuracy vs. Every # accuracy
n = 48
Above baseline
Below baseline
n.s.
***
Every
big circle
is
blue

39

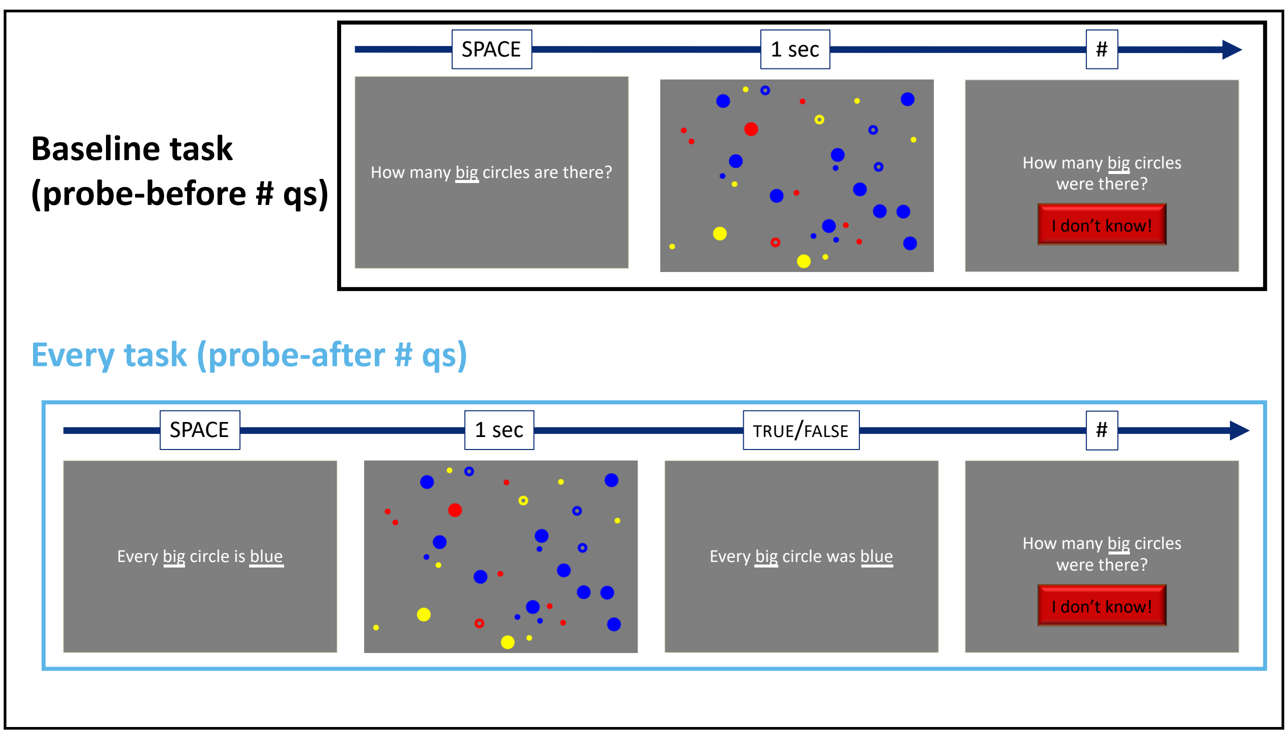

Every blue circle is big
How many {blue/big} circles were there?
Baseline # accuracy vs. Every # accuracy
n = 55
Above baseline
Below baseline
n.s.
**
Every
blue circle
is
big

40



Only big circles are blue
How many {big/blue} circles were there?
Baseline # accuracy vs. Only # accuracy
n = 48
Above baseline
Below baseline
**
***
Only
big circles
are
blue

41



Baseline task (probe-before # qs)
SPACE
1 sec
#
How many big circles are there?
How many big circles were there?
I don't know!
Every task (probe-after # qs)
SPACE
1 sec
TRUE/FALSE
#
Every big circle is blue
Every big circle was blue
How many big circles were there?
I don't know!

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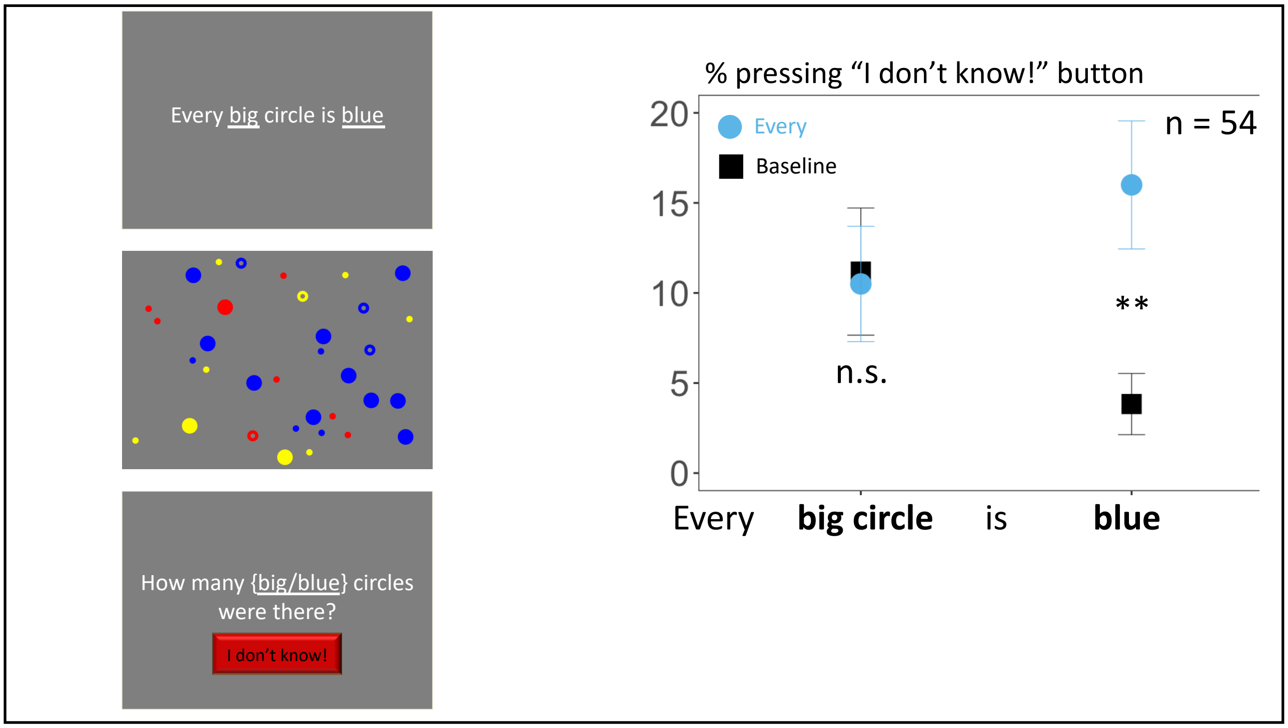
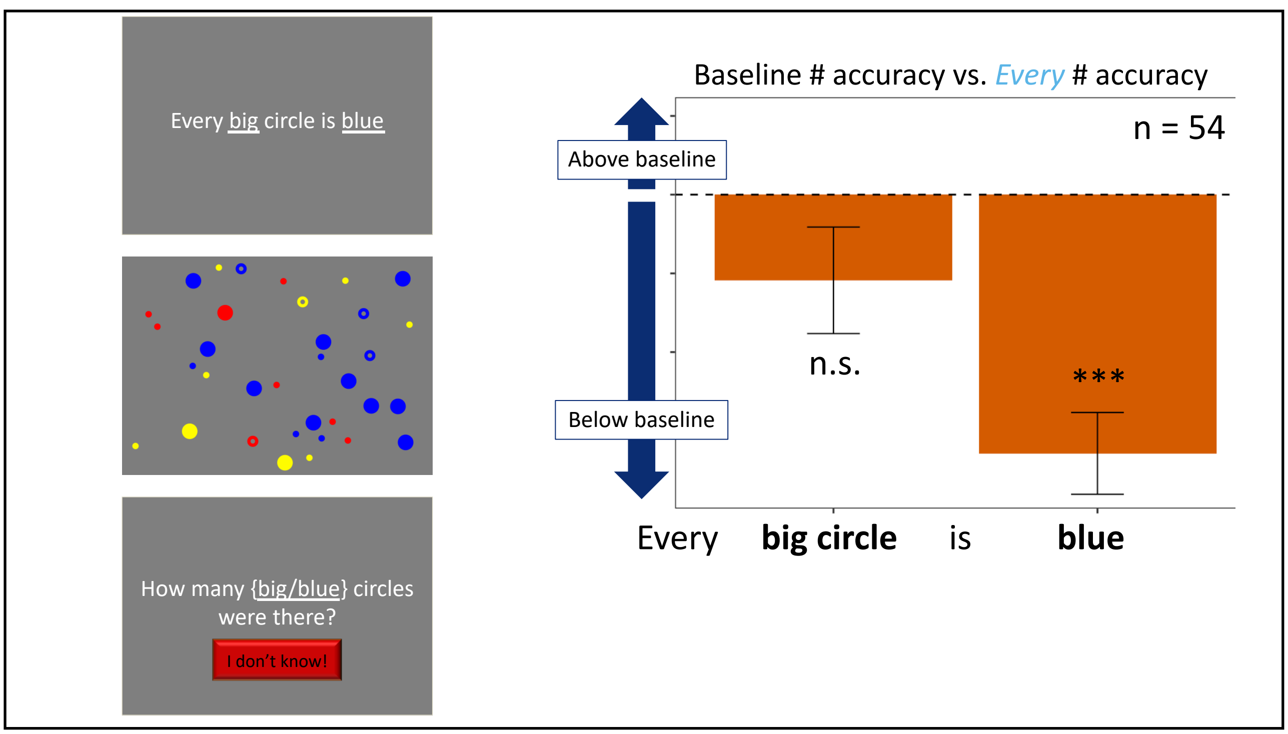


Every big circle is blue
How many {big/blue} circles were there?
I don't know!
% pressing "I don't know!" button
Every
Baseline
n = 54
n.s.
**
0
5
10
15
20
Every big circle is blue

43

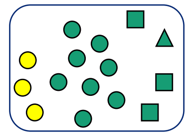


Every big circle is blue
How many {big/blue} circles were there?
I don't know!
Baseline # accuracy vs. Every # accuracy
n = 54
Above baseline
Below baseline
n.s.
***
Every big circle is blue

44

Every leads to
→ knowledge of 1st argument's cardinality
→ <u>no</u> knowledge of 2nd argument's cardinality

Only leads to
→ <u>no</u> knowledge of either set's cardinality



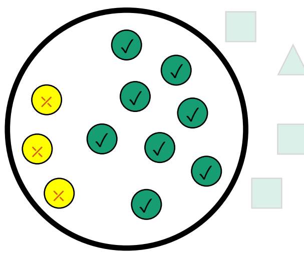
"Every circle is green"

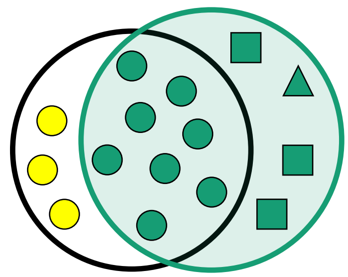
Zooming in on 1 group

Relating 2 groups

meaning → $\exists$CIRCLES & $\forall x{:}x \in$ CIRCLES[green(x)] | CIRCLES $\subseteq$ GREEN-THINGS

strategy →





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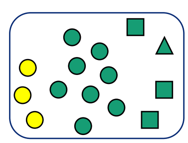
Every leads to
→ knowledge of 1st argument's cardinality
→ <u>no</u> knowledge of 2nd argument's cardinality

Only leads to
→ <u>no</u> knowledge of either set's cardinality



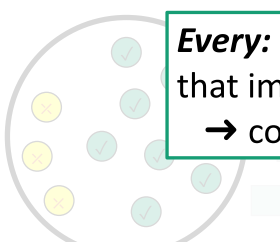
"Every circle is green"

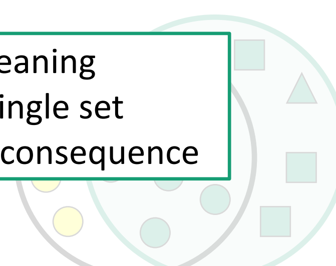
Zooming in on 1 group

Relating 2 groups

meaning → $\exists$CIRCLES & $\forall x{:}x \in$ CIRCLES[green(x)] | CIRCLES $\subseteq$ GREEN-THINGS

strategy →





Every: second-order meaning
that implicates only a single set
→ conservativity is a consequence

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All quantifiers are conservative!

(Barwise & Cooper, 1981; Higginbotham & May, 1981; Keenan & Stavi, 1986; a.o.)

Some ghosts are grey

Most ghosts are grey

Every ghost is grey

____ ghosts are grey

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All quantifiers are conservative!

(Barwise & Cooper, 1981; Higginbotham & May, 1981; Keenan & Stavi, 1986; a.o.)

A quantifier Q if conservative iff
$Q(X, Y) \leftrightarrow Q(X, X \cap Y)$

Some ghosts are grey ⟷ Some **ghosts** are grey **ghosts**

Most ghosts are grey ⟷ Most **ghosts** are grey **ghosts**

Every ghost is grey ⟷ Every **ghost** is a grey **ghost**

____ ghosts are grey ⟷ ____ **ghosts** are grey **ghosts**

The ghost with red hair has friends ⇹
The ghost **with red hair** has friends **with red hair**

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Possible non-conservative meanings?

Nonly ghosts are grey

≈ *not only ghosts are grey*

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Possible non-conservative meanings?

Nonly ghosts are grey

≈ *not only ghosts are grey*

Schmost ghosts are grey

≈ *ghosts outnumber grey things*

50

Possible non-conservative meanings?

Nonly ghosts are grey

≈ *not only ghosts are grey*

Schmost ghosts are grey

≈ *ghosts outnumber grey things*

Everynon ghost is grey

≈ *every non-ghost is grey*

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Possible non-conservative meanings?

A quantifier Q if conservative iff
$Q(X, Y) \longleftrightarrow Q(X, X \cap Y)$

Nonly ghosts are grey

≈ *not only ghosts are grey* ↮ *not only **ghosts** are grey **ghosts***

Schmost ghosts are grey

≈ *ghosts outnumber grey things* ↮ ***ghosts** outnumber grey **ghosts***

Everynon ghost is grey

≈ *every non-ghost is grey* ↮ *every non-**ghost** is a grey **ghost***

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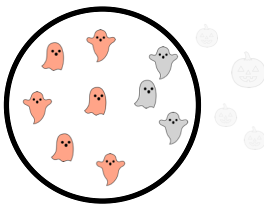
non-conservative
(and not attested as quantifier meanings)

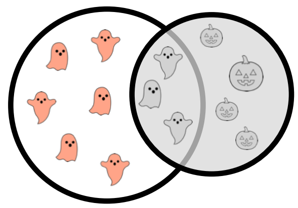
Schmost GHOSTS > GREY-THINGS
Equi GHOSTS = GREY-THINGS
Only GHOSTS ⊇ GREY-THINGS
Nonly GHOSTS ⊉ GREY-THINGS
Every GHOSTS ⊆ GREY-THINGS

Zoom in on
1 set

Relate
2 sets

Non-conservative meanings not stateable





Conservativity filter needed

e.g., Chierchia, 1995; Fox 2002; Ludlow, 2002; Sportiche, 2005; Romoli, 2015

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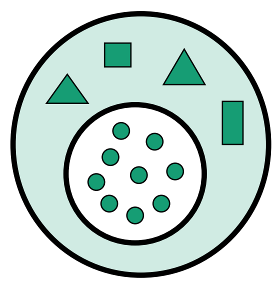
⟦**Each / Every circle is green**⟧ = TRUE *iff*



∀x : circle(x)[green(x)]
≈ for each thing that's a circle, it's green

¬∃x : circle(x)[¬ green(x)]
≈ there's no thing that's a circle but not green

CIRCLES ⊆ GREEN-THINGS
≈ the set of circles is a subset of the set of green things

CIRCLES = CIRCLES ∩ GREEN-THINGS
≈ the set of circles is identical to the set of green circles

∃CIRCLES & ∀x:x ∈ CIRCLES[green(x)]
≈ the members of the set of circles are each green

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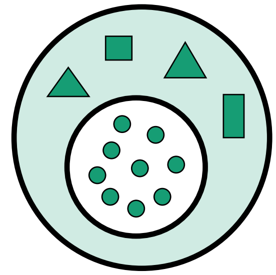
⟦**Each** / **Every** **circle is green**⟧ = TRUE *iff*



$\forall x : \text{circle}(x)[\text{green}(x)]$

≈ for each thing that's a circle, it's green

$\neg\exists x : \text{circle}(x)[\neg\, \text{green}(x)]$

≈ there's no thing that's a circle but not green

$\text{CIRCLES} \subseteq \text{GREEN-THINGS}$

≈ the set of circles is a subset of the set of green things

$\text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS}$

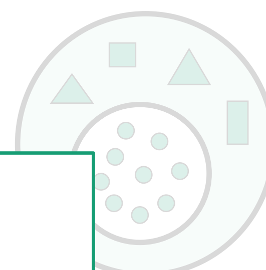
≈ the set of circles is identical to the set of green circles

$\exists\text{CIRCLES} \;\&\; \forall x{:}x \in \text{CIRCLES}[\text{green}(x)]$

≈ the members of the set of circles are each green

55

⟦Each / Every circle is green⟧ = TRUE *iff*



Bigger picture questions:

➜ Are lexical meanings invariant across people?

Yes! (at least some logical vocab)

➜ Are they structured or atomic?

Structured!

➜ If invariant & structured, how are they acquired?

Stay tuned!

≈ the members of the set of circles are each green

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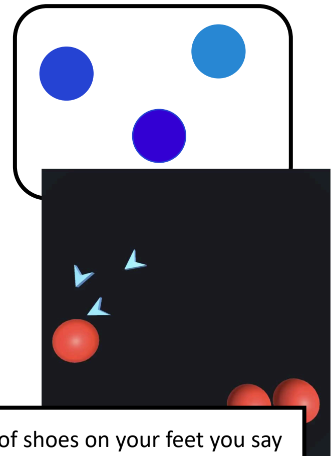
Ongoing projects

→ *Each* leads to knowledge of individual properties

→ Precursors of quantificational thought in infants

→ Corpus study of child-directed speech



Are we going to make a sandwich for **each** of your teddys?	**Every** time you get a pair of shoes on your feet you say they don't fit you
You pour some milk into **each** one of these cups	Trains shouldn't crash **every** 5 minutes, Dominic
In fact you have five fingers and five spiders; you could put one spider on **each** finger	You need to learn to like it on your belly instead of screaming **every** time, then you could learn to crawl!

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Thanks!

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& the members of the UMD AcqLab

Language at Maryland

NSF

James S. McDonnell Foundation

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