The psychological treatment of (universal) quantification

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\[ \text{[Each / Every circle is green]} = \]

(e.g., Barwise & Cooper 1981)
[Each / Every circle is green] = TRUE iff

\[ \forall x : \text{circle}(x) \land \text{green}(x) \approx \text{for each thing that's a circle, it's green} \]
\[ \neg \exists x : \text{circle}(x) \land \neg \text{green}(x) \approx \text{there's no thing that's a circle but not green} \]
\[ \text{CIRCLES} \subseteq \text{GREEN-THINGS} \approx \text{the set of circles is a subset of the set of green things} \]
\[ \text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS} \approx \text{the set of circles is identical to the set of green circles} \]
\[ \exists \text{CIRCLES} \land \forall x : x \in \text{CIRCLES} \land \text{green}(x) \approx \text{the members of the set of circles are each green} \]

"...I believe that the model-theoretic intension of a word has in principle nothing whatsoever to do with what goes on in a person's head when he uses that word"

-Dowty (1979)
[Each / Every circle is green] = TRUE iff

\[ \forall x : \text{circle}(x)[\text{green}(x)] \]
\[ \text{for each thing that's a circle, it's green} \]
\[ \neg \exists x : \text{circle}(x)[\neg \text{green}(x)] \]
\[ \text{there's no thing that's a circle but not green} \]

\text{CIRCLES} \subseteq \text{GREEN-THINGS}
\[ \text{the set of circles is a subset of the set of green things} \]
\text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS}
\[ \text{the set of circles is identical to the set of green circles} \]
\exists \text{CIRCLES} \& \forall x : x \in \text{CIRCLES}[\text{green}(x)]
\[ \text{the members of the set of circles are each green} \]
Each / Every circle is green = TRUE iff

First-order (categorizing individuals)

\[ \forall x : \text{circle}(x)[\text{green}(x)] \]
\[ \equiv \text{for each thing that’s a circle, it’s green} \]
\[ \neg \exists x : \text{circle}(x)[\neg \text{green}(x)] \]
\[ \equiv \text{there’s no thing that’s a circle but not green} \]

Second-order (implicating groups)

\[ \text{CIRCLES} \subseteq \text{GREEN-THINGS} \]
\[ \equiv \text{the set of circles is a subset of the set of green things} \]
\[ \text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS} \]
\[ \equiv \text{the set of circles is identical to the set of green circles} \]
\[ \exists \text{CIRCLES} \& \forall x : x \in \text{CIRCLES}[\text{green}(x)] \]
\[ \equiv \text{the members of the set of circles are each green} \]
### First-order (categorizing individuals)

**Each / Every circle is green**

\[
\forall x : \text{circle}(x)[\text{green}(x)] \quad \text{iff} \quad \neg \exists x : \text{circle}(x)[\neg \text{green}(x)]
\]

- for each thing that’s a circle, it’s green
- there’s no thing that’s a circle but not green

\[
\text{CIRCLES} \subseteq \text{GREEN-THINGS}
\]

\[
\exists \text{CIRCLES} & \quad \forall x : x \in \text{CIRCLES}[\text{green}(x)]
\]

- the set of circles is identical to the set of green circles
- the members of the set of circles are each green

### Second-order (implicating groups)

Does every’s meaning relate 2 sets or call for first-order quantification relativized to 1 set?

\[
\text{CIRCLES} \subseteq \text{GREEN-THINGS}
\]

- the set of circles is a subset of the set of green things

\[
\text{CIRCLES} = \text{CIRCLES} \cap \text{GREEN-THINGS}
\]

- the set of circles is identical to the set of green circles

\[
\exists \text{CIRCLES} & \quad \forall x : x \in \text{CIRCLES}[\text{green}(x)]
\]

- the members of the set of circles are each green

Each is mandatorily distributive (Vendler, 1962; Dowty, 1987)

- *Each (of the) student(s)*
- ?Every student
- All of the students

met at the bar / gathered
Bigger picture questions:

→ Are lexical meanings invariant across people?
→ Are they structured or atomic?
→ If invariant & structured, how are they acquired?
"Each/Every circle is green"

Categorizing individuals
∀x : circle(x)[green(x)]

Relating groups
CIRCLES ⊆ GREEN-THINGS

Interface transparency
(Pietroski et al. 2009; Lidz et al. 2011)

Bias to use verification strategy
that directly implements meaning
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Bias to use verification strategy
that directly implements meaning

Categorizing individuals
\( \forall x : \text{circle}(x)[\text{green}(x)] \)

Relating groups
\( \text{CIRCLES} \subseteq \text{GREEN-THINGS} \)

“Each/Every circle is green”

Representing groups:
good estimate of summary statistics
(number, average size, center of mass, ...)
Ariely 2001; Cohng & Treisman 2003; Feigenson et al. 2004; Burr & Ross 2008; Alvarez 2011; ao

Measuring #-knowledge: accuracy (\( \beta \))

\[ y = \alpha x^\beta \]

\( \beta=1.2 \)
\( \beta=1.1 \)
\( \beta=1 \)
\( \beta=0.9 \)
\( \beta=0.8 \)

(Stevens 1964; Krueger 1984; Odic et al. 2016; etc.)
Measuring #-knowledge: accuracy ($\beta$)

\[ y = \alpha x^\beta \]

(Stevens 1964; Krueger 1984; Odic et al. 2016; etc.)

Measuring #-knowledge: precision ($\sigma$)

\[ \beta = .8 \quad \sigma = .1 \]

(Laming 1997; Odic et al. 2016; etc.)
Measuring #-knowledge: precision ($\sigma$)

Average Numerical Response vs. Number of dots on screen

- $\beta = 0.8$
- $\sigma = 0.2$

(Laming 1997; Odic et al. 2016; etc.)

Measuring #-knowledge: precision ($\sigma$)

Average Numerical Response vs. Number of dots on screen

- $\beta = 0.8$
- $\sigma = 0.3$

(Laming 1997; Odic et al. 2016; etc.)
Parameter fits (# estimation model)

- (Each/All) of the big dots are blue
- How many big dots were there?

Better performance

n=24

accuracy ($\beta$) | precision (1-\(\sigma\))

All | Each

- n.s.
- **

22
(Each/All) of the big dots are blue

How many big dots were there?

Parameter fits (# estimation model)

Better performance

First question accuracy

n=24

n.s.

Parameter fits (# estimation model)

Better performance

n=24

n.s.
Parameter fits (# estimation model)

<table>
<thead>
<tr>
<th>accuracy (β)</th>
<th>precision (1-σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Every</strong></td>
<td>***</td>
</tr>
<tr>
<td><strong>Each</strong></td>
<td></td>
</tr>
</tbody>
</table>

First question accuracy

Better performance

n=24

Parameter fits (# estimation model)

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</tr>
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<tbody>
<tr>
<td><strong>All of the</strong></td>
<td>n.s.</td>
</tr>
<tr>
<td><strong>Every</strong></td>
<td>***</td>
</tr>
</tbody>
</table>

Better performance

n=24

How many big dots were there?
Every leads to superior → cardinality estimation in adults

“Each/Every circle is green”

Categorizing individuals
∀x : circle(x) [green(x)]

Relating groups
CIRCLES ⊆ GREEN-THINGS

meaning →

strategy →

{Each/Every} circle is blue

Distance from tap to actual set center

Each
Every

More error

n = 55 (so far)

Where was the middle of the circles?
Is each/every circle blue?

Where was the middle of the circles?

Correctly answering ‘no’

<table>
<thead>
<tr>
<th>age</th>
<th>Each</th>
<th>Every</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4-5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5-6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6-7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7-8</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

n = 151
Ages: 3;2 - 7;11
Mean age = 5;4

Distance from tap to actual set center

<table>
<thead>
<tr>
<th>error (millimeters)</th>
<th>Each</th>
<th>Every</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

n = 109
Ages: 3;2 - 7;11
Mean age = 5;8

More error

* *
Is each/every circle blue?

Where was the middle of the circles?

Each

Every

\[ n = 109 \]

Ages: 3;2 - 7;12

Mean age = 5;8

Not just a learned strategy!

Distance from tap to actual set center

\[ \forall x : \text{circle}(x) \rightarrow \text{green}(x) \]

“Each/Every circle is green”

Every leads to superior

\[ \rightarrow \text{cardinality estimation in adults} \]

\[ \rightarrow \text{memory for center of mass in adults} \]

\[ \rightarrow \text{memory for center of mass in kids} \]

Each leads to superior

\[ \rightarrow \text{memory for individual properties (ongoing)} \]
**Every** leads to superior
→ cardinality estimation in adults
→ memory for center of mass in adults
→ memory for center of mass in kids

**Each** leads to superior
→ memory for individual properties (ongoing)

"Each/Every circle is green"

Categorizing individuals
∀x : circle(x)[green(x)]

Relating groups
CIRCLES ⊆ GREEN-THINGS

**Each:** first-order meaning
**Every:** second-order meaning

"Each/Every circle is green"

Categorizing individuals
∀x : circle(x)[green(x)]

Relating groups
CIRCLES ⊆ GREEN-THINGS

**Each:** first-order meaning
**Every:** second-order meaning

→ But which one?
"Every circle is green"

Zooming in on 1 group

\[ \exists \text{CIRCLES} \& \forall x: x \in \text{CIRCLES}[\text{green}(x)] \]

Relating 2 groups

\[ \text{CIRCLES} \subseteq \text{GREEN-THINGS} \]

Baseline task (probe-before # qs)

How many big circles are there?

Every task (probe-after # qs)

Every big circle is blue

How many big circles were there?
Every **big circle is blue**

Zoom in on 1 group

Relate 2 groups

Accuracy on # question

1st argument

2nd argument

Baseline

Every

Baseline

Above baseline

Below baseline
Every big circle is blue

How many (big/blue) circles were there?

Baseline # accuracy vs. Every # accuracy

Above baseline

Below baseline

Every big circle is blue

n = 48

n.s. ***

Every blue circle is big

How many (blue/big) circles were there?

Baseline # accuracy vs. Every # accuracy

Above baseline

Below baseline

Every blue circle is big

n = 55

n.s. **
Only big circles are blue

How many big/blue circles were there?

Baseline # accuracy vs. Only # accuracy

Above baseline

Below baseline

n = 48

Only big circles are blue

Baseline task (probe-before # qs)

How many big circles are there?

How many big circles were there?

I don’t know!

Every task (probe-after # qs)

Every big circle is blue

Every big circle was blue

How many big circles were there?

I don’t know!
Every \textcolor{red}{\textbf{big}} circle \textcolor{blue}{\textbf{is}} \textcolor{blue}{\textbf{blue}}

How many \textcolor{red}{\textbf{big/blue}} circles were there?

% pressing “I don’t know!” button

\begin{itemize}
\item Every
\item Baseline
\end{itemize}

n = 54

\begin{itemize}
\item n.s.
\item **
\end{itemize}

Below baseline

Above baseline

Baseline # accuracy vs. \textcolor{red}{\textbf{Every}} # accuracy

\begin{itemize}
\item n.s.
\item ***
\end{itemize}

How many \textcolor{red}{\textbf{big/blue}} circles were there?

Every \textcolor{red}{\textbf{big circle}} \textcolor{blue}{\textbf{is}} \textcolor{blue}{\textbf{blue}}
**Every** leads to
- knowledge of 1st argument’s cardinality
- no knowledge of 2nd argument’s cardinality

**Only** leads to
- no knowledge of either set’s cardinality

“Every circle is green”

**Every** leads to
- knowledge of 1st argument’s cardinality
- no knowledge of 2nd argument’s cardinality

**Only** leads to
- no knowledge of either set’s cardinality

“Every circle is green”

**Every**: second-order meaning that implicates only a single set
- conservativity is a consequence
All quantifiers are conservative!

(Barwise & Cooper, 1981; Higginbotham & May, 1981; Keenan & Stavi, 1986; a.o.)

Some ghosts are grey
Most ghosts are grey
Every ghost is grey
___ ghosts are grey

A quantifier Q if conservative iff
Q(X, Y) ⇔ Q(X, X∩Y)

The ghost with red hair has friends ⇔
The ghost with red hair has friends with red hair

Some ghosts are grey ⇔ Some ghosts are grey ghosts
Most ghosts are grey ⇔ Most ghosts are grey ghosts
Every ghost is grey ⇔ Every ghost is a grey ghost
___ ghosts are grey ⇔ ___ ghosts are grey ghosts
Possible non-conservative meanings?

Nonly ghosts are grey
≈ not only ghosts are grey

Schmost ghosts are grey
≈ ghosts outnumber grey things
Possible non-conservative meanings?

Nonly ghosts are grey
≈ not only ghosts are grey

Schmost ghosts are grey
≈ ghosts outnumber grey things

Everynon ghost is grey
≈ every non-ghost is grey

A quantifier Q if conservative iff
Q(X, Y) ↔ Q(X, X∩Y)
non-conservative
(and not attested as quantifier meanings)

Schmost
Equi
Only
Nonly
Every

GHOSTS > GREY-THINGS
GHOSTS = GREY-THINGS
GHOSTS ⊇ GREY-THINGS
GHOSTS ⊈ GREY-THINGS
GHOSTS ⊆ GREY-THINGS

Zoom in on
1 set

Relate
2 sets

Conservativity filter
needed
e.g., Chierchia, 1995; Fox 2002;
Ludlow, 2002; Sportiche, 2005;
Romoli, 2015

⟦Each / Every circle is green⟧ = TRUE iff

∀x : circle(x)[green(x)]
≈ for each thing that’s a circle, it’s green
¬∃x : circle(x)[¬ green(x)]
≈ there’s no thing that’s a circle but not green
CIRCLES ⊆ GREEN-THINGS
≈ the set of circles is a subset of the set of green things
CIRCLES = CIRCLES ∩ GREEN-THINGS
≈ the set of circles is identical to the set of green circles
∃CIRCLES & ∀x:x ∈ CIRCLES[green(x)]
≈ the members of the set of circles are each green
[[Each / Every circle is green]] = TRUE iff

\[ \forall x : \text{circle}(x) [\text{green}(x)] \]
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\[ \approx \text{the members of the set of circles are each green} \]

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**Bigger picture questions:**

- Are lexical meanings invariant across people?
  Yes! (at least some logical vocab)

- Are they structured or atomic?
  **Structured!**

- If invariant & structured, how are they acquired?
  **Stay tuned!**
Ongoing projects

- *Each* leads to knowledge of individual properties
- Precursors of quantificational thought in infants
- Corpus study of child-directed speech

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<table>
<thead>
<tr>
<th>Are we going to make a sandwich for each of your teddys?</th>
<th>Every time you get a pair of shoes on your feet you say they don’t fit you</th>
</tr>
</thead>
<tbody>
<tr>
<td>You pour some milk into each one of these cups</td>
<td>Trains shouldn’t crash every 5 minutes, Dominic</td>
</tr>
<tr>
<td>In fact you have five fingers and five spiders; you could put one spider on each finger</td>
<td>You need to learn to like it on your belly instead of screaming every time, then you could learn to crawl!</td>
</tr>
</tbody>
</table>

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Thanks!

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